Damage Tolerance Assessment Using Stress Intensity Threshold Values

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Abstract

This tech brief outlines a method of employing fracture mechanics as a means of cross checking high cycle fatigue (HCF) predictions. The technique enables an engineer to evaluate the robustness of a design against accumulated HCF fatigue damage in a notch using a damage tolerance approach. The intent is to help increase the designer's understanding of the mechanistic processes involved in crack initiation and provide a tool in assessing the likelihood of a crack being arrested from a stress concentration feature as it transitions from a micro to a macro crack .

The limitations of linear elastic fatigue mechanics (LEFM) in the near threshold crack growth regime are discussed and how it relates to fatigue initiation. The J-integral is employed in evaluating loading scenarios where local and net section plasticity is present. The CINT macro in ANSYS version 11 is employed to perform the integration.

Damage Tolerance Assessment

Evaluating the HCF capability of a component is typically a stress based approach that employs an endurance limit taken from an S-N curve at 10E6 cycles. This endurance limit is adjusted to account for mean stress, surface quality, reliability, etc. The evaluation is then made by comparing the alternating stress loading against the adjusted endurance limit. For micro cracks (cracks small relative to the alloy's microstructure) the endurance limit is the controlling parameter. The stress intensity threshold value, however, is the parameter to consider when determining whether or not a macro crack will propagate.

To confirm a stress based HCF evaluation using a damage tolerance approach, an initial crack is assumed in the limiting feature that equals the transition length from a micro to macro crack. Using this crack length, the stress intensity range is calculated and compared to the threshold stress intensity value for the alloy. A stress intensity range below the threshold (ΔK_{th}) would indicate that the flaw would be arrested and not propagate. It would be anticipated that if the two different approaches are valid they would tend to approach the same answer and corroborate the other in the transition from a micro to macro crack. A positive margin with a stress based approach predicts infinite life for the HCF loading and the damage tolerance method would predict an arrested crack.

Computing Stress Intensity Values

For LEFM, the maximum stress intensity value can be calculated from equation 1. S_{max} is the maximum gross stress in the limiting load path, and Q is the geometry factor which is governed by the crack size "a" and the net section geometry. Q can be obtained by referencing any standard handbook on fracture mechanics.[1]

Eq. 1.0
$$K_{\max} = S_{\max} K_t Q \sqrt{\pi a}$$

Applying the K_t value assumes that the flaw is within the process zone of the stress concentration feature. The process zone is typically in the neighborhood of 20 percent of the notch or fillet radius of the geometric stress concentration. [2]

The stress intensity range is then determined from using the stress ratio (R) of the HCF loading as shown in equation 2. R is the ratio of the minimum to maximum stress

Eq. 2.0
$$\Delta K = K_{\max} (1 - R)^{\gamma}$$

Finally, the stress intensity range (ΔK) is compared to ΔK_{th} of the alloy for the given R ratio. A ΔK below the threshold value indicates that the crack would not propagate. Research on the effects of R ratio on ΔK_{th} can be found in the literature referenced below.[3 & 4]

The critical parameter required to undertake this evaluation is the value for the appropriate length of the crack as it transitions from a micro to macro

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[1] An excellent resource that covers both fatigue and fracture mechanics is Norman Dowling's *Mechanical Behavior of Materials*, Prentice Hall, ISBN 0-13-579046-8

[2] Ibid., p.299

[3] Schmidt RA, Paris PC. Threshold for fatigue crack propagation and the effects of load ratio and frequency. In: Progress in flaw growth and fracture toughness testing, ASTM STP 536. Philadelphia, PA: American Society for Testing and Materials; 1973. p. 79–94.

[4] Suresh S, Ritchie RO. On the influence of environment on the load ratio dependence of the fatigue threshold in pressure vessel steel. Eng Fract Mech 1983;18:785–800

crack. In addition, the limitations of the fracture mechanics methodology need to be understood so that the approach is properly applied.

Lower Bound Thresholds Values for Steel

Work undertaken by Barsom and Rolfe provides a reasonable lower bound stress intensity threshold as a function of the R ratio.[5] For the various steels investigated, the lower limit of the scatter provides the following data:

For R ratios $\leq$ 0.17

Eq. 3.0 $$\Delta K_{th} = 5.5 ksi\sqrt{in}$$

For R ratios $\geq$ 0.17

Eq. 4.0 $$\Delta K_{th} = 6.4(1 - 0.85R) ksi\sqrt{in}$$

Equations 3.0 and 4.0 represent a reasonable worst case scenario for a wide range of steels, but lower threshold values do exist for higher strength alloys.

Classifications of Crack Size

The size of the assumed flaw is important in obtaining a valid damage tolerance assessment. The growth rate of near threshold cracks will generally be underestimated using facture mechanics when the criteria for a macro crack is not met. It is essential, therefore, that the assumed initial flaw size is large enough to meet the requirements in the damage tolerance assessment.

Crack size can be classified as it relates to the alloy's microstructure, size of the near tip plasticity zone, and the ability to detect it with a given inspection technique. This classification approach is concerned with the relative size of a crack with respect to mechanical aspects of the particular alloy and/or component. Small cracks can exist that are large enough to fall under the purview of fracture mechanics but exhibit anomalies in propagation rates due to environmental factors. These flaws are typically referred to as chemically small cracks. Propagation of this nature can be driven by energy activation mechanisms rather than damage accumulated on limiting shear slip planes. Additionally, oxidation on the crack faces may effect the cyclic crack closure behavior.

Cracks that are small relative to the microstructure are typically less than 5 to 10 times the alloy's grain size and are often referred to as micro-cracks. A grain is formed by the uniform alignment crystalline lattice structures characterizing the given alloy. The front of a micro-crack is characterized by single shear slip bands aligned with the primary slip system in the grain. This propagation results in a zig-zag path and is referred to as Stage or Regime I crack growth. In contrast to a macro crack which will tend to exhibit duplex shear bands at the crack front resulting in striations on the surface of the crack face.

Inspection techniques are typically limited to detecting crack sizes of approximately 0.3 mm, although this is case dependent. Cracks of much smaller size can be detected on a regular basis with techniques such as fluorescent dye penetrant.

Plane Strain Versus Plane Stress Conditions

If the near tip plasticity zone engulfs the entire crack then LEFM may not apply due to the crack being small compared to the surrounding K-field. LEFM is a technique that relates the gross stress field to the behavior of the plastic zone at the crack front. The K or stress intensity field is essientally a transfer function between the gross stress field and the crack front. If the crack is completely engulfed in the near tip plasticity zone, the plasticity creates a plane stress condition in the K-field rather than a plane strain condition that the stress intensity data is based on.

The minimum crack size for a plane strain condition to exist is given in equation 5.0.

Eq. 5.0 $$a \geq 2.5\left(\frac{K}{\sigma_o}\right)^2$$

If this criteria is not met, then plane stress will characterize the local deformation behavior in the K field. If the estimated transition crack length from a micro to macro crack is less than the crack length in equation 5.0, then the cracked is engulf in the plastic zone and plane stress behavior will dominate.

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[5] Dowling NE, Mechanical Behavior of Materials, Prentice Hall, 1993, p. 470. Original work by Barsom and Rolfe

LEFM, however, can also be applicable under plane stress conditions as long as the crack length is 8 times greater than the radius of the cyclic plastic zone. The plastic zone radius for a plane stress condition can be computed from equation 6.0.[6]

Eq. 6.0
$$r_{o\sigma} = \frac{1}{2\pi}\left(\frac{\Delta K}{2\sigma_o}\right)^2$$

where σ_o is the 0.02% cyclic yield strength.

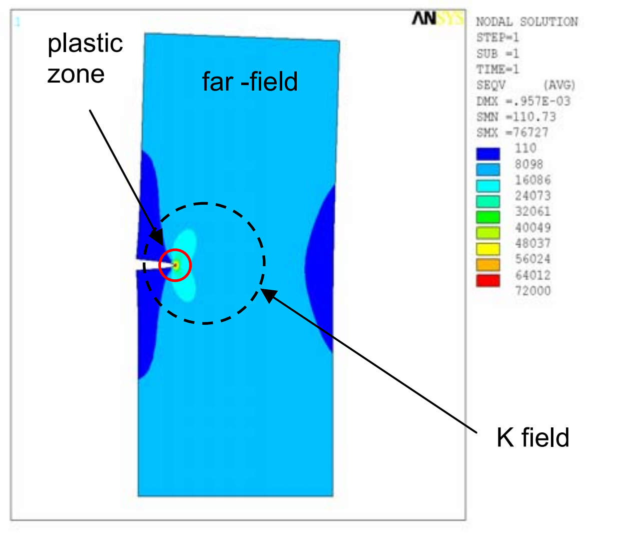

Figure 1 - K field

Characteristics of Crack Growth Behavior

The life of a crack can be characterized into three regimes. As shown in Figure 2, Regime I is where the stress intensity range (ΔK) is below the threshold value and where crack growth does not occur or the change in the low rate of crack growth approaches a step function. The second regime is where the stress intensity range is great enough that the crack propagates at a rate that follows Paris' equation.[7] The third regime is when the life of the crack has grown to the point that the stress intensity range approaches the fracture toughness of the alloy (K_{IC}). In this regime the crack growth increases rapidly to rupture.

As stated earlier, the growth rate of near threshold cracks in Regime I will generally be underestimated using fracture mechanics if the criteria for a macro crack length is not met. The growth rate will also tend to be underestimated using LEFM if the crack length is not 8 times the plastic zone radius at the crack front. Employing fracture mechanics in a HCF damage tolerance evaluation requires that the assumed minimum crack is sufficiently large for the approach to apply .

For micro cracks the endurance limit is the controlling parameter to determine fatigue initiation while the stress intensity threshold value is the limiting parameter for evaluating the propagation of macro cracks.

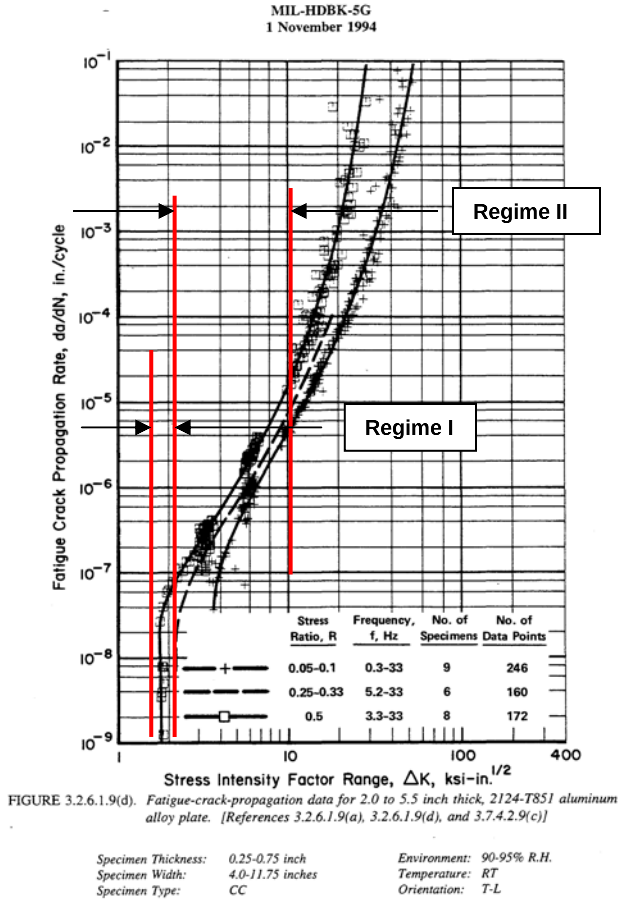

FIGURE 3.2.6.1.9(d). *Fatigue-crack-propagation data for 2.0 to 5.5 inch thick, 2124-T851 aluminum alloy plate. [References 3.2.6.1.9(a), 3.2.6.1.9(d), and 3.7.4.2.9(c)]*

Specimen Thickness: *0.25-0.75 inch*
Specimen Width: *4.0-11.75 inches*
Specimen Type: *CC*
Environment: *90-95% R.H.*
Temperature: *RT*
Orientation: *T-L*

Figure 2 - Crack Growth Regimes

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[6] S.Suresh, Fatigue of Materials, Cambridge, 1998, p. 306

[7] $\frac{da}{dN} = C(\Delta K)^m$ where da/dN is the rate of crack propagation

Estimating the Minimum Macro Crack Size

Estimating the size of a crack transitioning from a micro to a macro flaw (a_o) can be done by employing equation 7.0.

Eq. 7.0 $$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{Q\Delta\sigma_e'}\right)^2$$

where:

ΔK_{th} is the threshold value and $\Delta\sigma_e$ is the endurance limit range that have been adjusted to the appropriate R ratio. Q is the geometry factor associated with the crack and net section geometry.

Mean Stress Model for Stress Based Evaluations

The Goodman mean stress model is typically employed in adjusting the endurance limit for a stress ratio greater than -1. Other mean stress model, however, are available. Due to variations in the mean stress model that one might employ, it is prudent to check the length of the transition crack a_o at both the appropriate R ratio and at R of -1 and use the limiting case.

For the Goodman mean stress model the adjusted endurance limit is provided in equation 8.0.

Eq. 8.0 $$\sigma_e' = \sigma_e\left(1-\frac{\sigma_m}{\sigma_u}\right)$$

Example of Damage Tolerance Assessment

A stepped bar is employed to illustrate the process of cross checking the robustness of a stress based infinite life evaluation with the facture mechanics damage tolerance approach. The bar used in this analysis is shown in Figure 3.

Hot rolled 4340 is the chosen material and the average strain life properties have been taken from the SAE J1099 technical report, revised June of 1998. A factor of 0.753 has been applied to the strain life curve to estimate lower 3σ properties. The same factor has been applied to the ultimate strength to provide a lower bound Goodman mean stress model. The endurance limit is obtained from the strength portion of the strain life equation at 10E6 cycles. The average endurance limit is 35.2 ksi. The estimated lower 3σ value is 26.5 ksi.

The cross check is provided at three different stress ratios. The first check is at a R ratio of -1. The second at R = 0.76 and the third at 0.95.

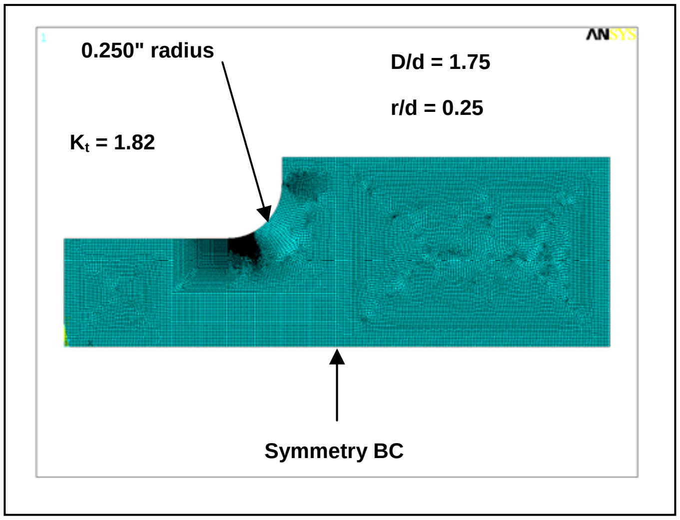

Figure 3 - Stepped Bar Model

Process Zone and Stress Concentration Blunting

The blunting of stress concentrations has been demonstrated from a wealth of empirical data. This occurs when the notch radius becomes small enough that the effective stress concentration is significantly lower than the elastic K_t value. When this occurs the HCF life is typically based on K_f, the effective stress concentration, rather than the elastic K_t.

Although no universally accepted theory is available that explains this blunting phenomena, it is generally agreed that the stress gradient is the primary parameter effecting it. The step radius, in this example, is relatively large compared to the net section and therefore approximately 30 percent of the net section is above the gross section loading of 10 ksi. No significant blunting effect would be expected with this low stress gradient.

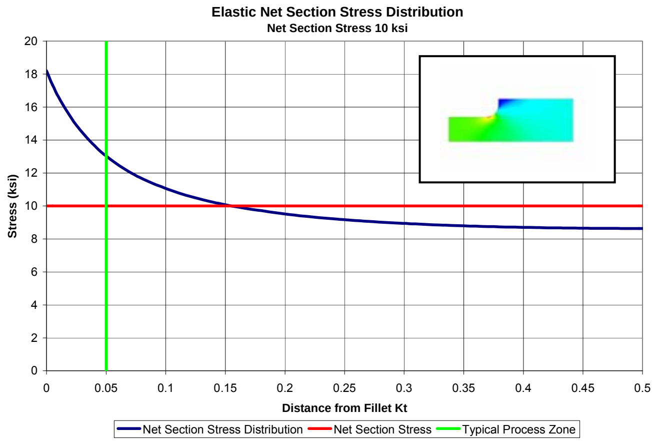

Figure 4 - Net Section Stress Distribution

The process zone, which is typically considered 20 percent of the notch radius, is also greater than the assumed damage tolerant cracks used in the cross check. The stress concentration effects of the radius, therefore, will be applicable to the stress intensity calculations as provided in equation 1.0.

Allowable HCF Alternating Stress

The ultimate strength is the maximum mean corresponding to a zero allowable alternating stress for the Goodman mean stress model. The estimated lower 3σ ultimate strength is 87.5 ksi for the hot rolled 4340. The allowable alternating stress for infinite life at the three different stress ratios is shown below.

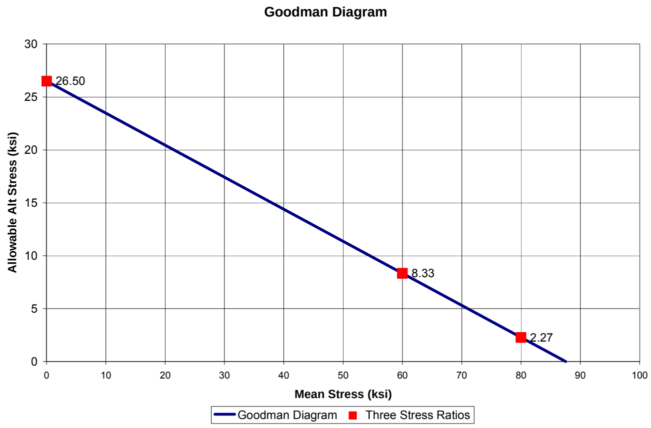

Figure 5 - Allowable Alternating Stress

The values for the adjusted endurance limits for the three stress ratios are also provided in Table 1.

Table 1 - Stress Ratios and σ'_e

σ_{mean}	σ'_e	R_{ratio}
ksi	ksi	
0	26.50	-1.00
60	8.33	0.76
80	2.27	0.95

Minimum Macro Crack Size

Using equations 4.0 and 7.0, the minimum macro size can be calculated. The value of Q in equation 7.0 is 1.12. The results are provided in Table 2.

An additional check is made on the minimum crack size for the R ratio of -1.0. Since no net section plasticity will occur, LEFM can be employed for this evaluation. In order for LEFM to apply in under plane stress conditions, however, the size of the crack should be 8 times larger than the radius of the *cyclic* plastic zone at the crack front.

Table 2 - Minimum Crack Lengths

R ratio	ΔK_{th}	a
	ksi-in$^{0.50}$	in
-1.00	5.50	0.0027
0.76	2.27	0.0047
0.95	1.23	0.0187

The cyclic plane stress plastic zone can be calculated from equation 6.0. The equation is based on the kinematic hardening characteristics of ductile metallic alloys.

The proportional limit is approximately 45 ksi for the hot rolled 4340 and therefore the kinematic yield surface (e.g., 2σ_o) is 90 ksi. The estimated radius of the plane stress crack tip plastic zone is 0.0006 inches. For the R ratio of -1, the crack length required for LEFM to be applicable is 0.0048 inches.

For this loading condition the compressive portion of the cycle is assumed to have no effect. This is reasonable based on the logic that the crack closes under compression and therefore no longer acts as a crack. In more ductile material, however, the compressive portion of the load may contribute to crack growth so this approach is not universally applicable.

For the R ratios of -1.0 and 0.76 an assumed crack size of 0.005 inches is used for the damage tolerance approach. A crack size of 0.020 inches is used for the 0.94 R ratio. Since the mean stress for the R ratios of 0.76 and 0.95 create plasticity in the notch, the J integral is computed employing the CINT macro in ANSYS version 11 rather using classical LEFM calculations.

Damage Tolerance Cross Check

- Loading Scenario R = -1.0

The compressive portion of the R ratio loading of -1 is assumed to have no effect resulting in the effective stress intensity range equally the maximum stress intensity value. Equation 9.0 is employed to calculate K_{max} where 26.5 ksi is the concentrated fillet stress.

Eq. 9.0 $$\Delta K = K_{\max} = 26.5 \times 1.12 \times \sqrt{\pi \times 0.005}$$

The value for ΔK is 3.72 ksi-in $^{0.50}$ which is less than the minimum threshold value of 5.5, indicating that no crack propagation would be expected from the alternating notch stress of 26.5 ksi. This 32 percent positive margin, based on the stress intensity threshold, tends to corroborate the infinite life prediction from the Goodman mean stress model. This is a relatively high margin against the lower bound Goodman, but the assumption that the compressive portion of the loading having no effect on the crack may also be somewhat non-conservative.

- Loading Scenario R = 0.76

The loading scenarios with R ratios of 0.76 and 0.95 create plasticity in the notch. A 2D ANSYS finite element model was created which employs PLANE183 elements and the CINT macro to account for this plasticity. The CINT macro, a feature first introduced in version 11, automatically performs the J integration. Unlike the approach which employs singular quarter mid nodes and uses the crack tip displacement field to calculate the stress intensity factor, the CINT macro is highly mesh density sensitive.

The mesh density employed for the stepped bar model is shown in Figure 6.

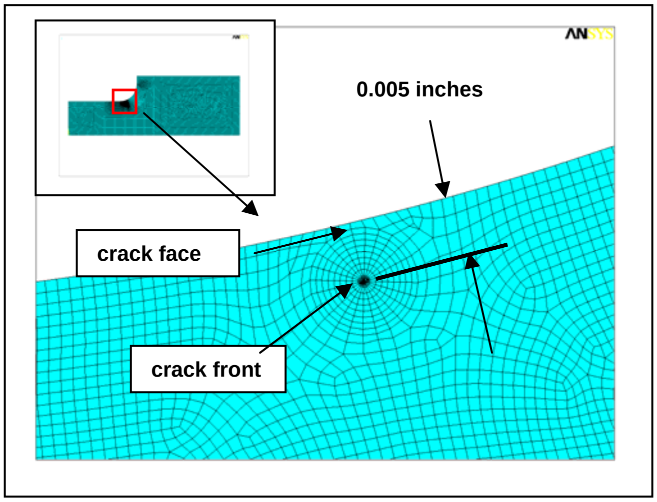

Figure 6 - Mesh Density for J Integral Evaluation

A kinematic hardening material model was used to simulate the plasticity in the notch. The true stress-strain curve was generated from the strain-life parameters documented in the SAE technical report J1099 using a Ramberg-Osgood model. The J-integral was converted to an equivalent stress intensity value using the relationship in equation 9.0[8]

Eq. 9.0 $$J = \frac{K^2}{E}$$

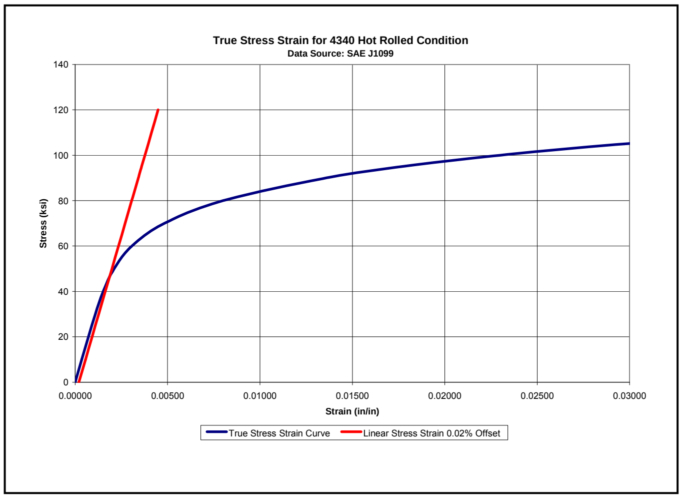

Figure 7 - True Stress-Strain Curve

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[8] This relationship has been employed due to the K field being governed by plane stress rather than plane strain.

For plane strain $J = \frac{(1-\nu^2)}{E} K^2$

Table 3 - R ratio Loading of 0.76

Mean Stress 60 ksi		
K_{max}	10.37	ksi-in$^{0.50}$
K_{min}	7.92	ksi-in$^{0.50}$
ΔK	2.45	ksi-in$^{0.50}$
Margin	-6.53%	

The 6.53 percent negative margin against the lower bound ΔK_{th}, of 2.27 ksi-in $^{0.50}$ provides a reasonable corroboration with the stress based fatigue analysis. The designer, however, should also keep in mind that although the high cycle loading may not accumulate damage, the life of the component is not infinite for this loading scenario. The maximum stress range for infinite life is 53 ksi. The major cycle of this loading scenario exceeds this value in addition to the mean stress effect. A strain-life fatigue evaluation would have to be undertaken to estimate the finite life of the part under this loading condition.

- Loading Scenario R = 0.95

The J integral ANSYS solution was obtained with crack size of 0.02 inches to simulate the transition length to a macro crack.

Table 4 - R ratio Loading of 0.95

Mean Stress 80 ksi		
K_{max}	27.35	ksi-in$^{0.50}$
K_{min}	25.97	ksi-in$^{0.50}$
ΔK	1.38	ksi-in$^{0.50}$
Margin	-12.2%	

Once again, the negative 12.2 percent margin against the lower bound ΔK_{th}, of 1.23 ksi-in $^{0.50}$ indicates relatively good correlation with the Goodman mean stress model. As with the previous scenario, the 80 ksi mean stress loading would create a finite life condition even if no damage was accumulated with the 2.27 ksi high cycle alternating stress.

Figure 8 provides a plot of the transition macro crack predictions against the lower bound stress intensity threshold values reported by Barsom.

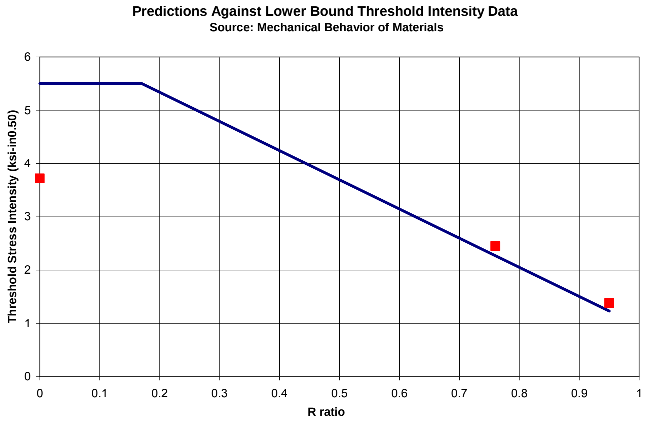
Predictions Against Lower Bound Threshold Intensity Data
Source: Mechanical Behavior of Materials

Figure 8 - Damage Tolerance Cross Check

Summary

The damage tolerance approach can be a means of determining whether or not the mean stress model and/or data employed in a HCF stress based evaluation is consistent with threshold fracture mechanics. Damage tolerance predictions that do not provide reasonable engineering correlation with the stress based evaluation give the designer good reason to either review the fatigue data employed in the stress analysis and/or address any potential short comings in the design.

Unlike low cycle fatigue scenarios, failure to meet the endurance limit under HCF conditions typically results in a very limited component or product life. This is usually due to the frequency content of most high cycle fatigue loading conditions. A damage tolerance cross check can provide the designer with valuable information regarding margin and additional insight into improving a design.

Further information regarding Integrated Systems Research, Inc. can be obtained at www.isrtechnical.com