Designing Load Path Transitions Using Stress Intensity Factors

Tech Brief 130301 F

Integrated Systems Research, Inc.

March, 2013

steve.carmichael@isrtechnical.com

Abstract:

When structural fatigue issues occur they are typically associated with regions of load path transitions. Transitions can take on many different forms, but the feature common to them is the local concentration of stress created by an interruption of the stress field. This concentration can facilitate the accumulation of fatigue damage under cyclical loading and eventually crack initiation.

This tech brief outlines a design technique for optimizing local transition features by minimizing the disruption of the stress field. The approach uses crack fronts as probes into the stress field. The resulting stress intensity factors are used to assess the behavior of the transition stress field from which local transition features can be optimally located and sized. The optimization is based on the alignment of the transition feature with respect to the maximum first principle stress. With the latest ANSYS 14.5 release, probing a stress field in complex geometry has become extremely efficient due to the ease at which elliptical cracks can be inserted into the base geometry.

This technique, however, can also be used with geometry in a primitive state to guide the detail design towards an optimal solution. This white paper provides several simple illustrative examples of how this design technique can be used. The approach can be extrapolated to more complex design challenges.

Background:

Tech Brief 120701 F outlines a technique which employs stress intensity factors using basic or primitive load path geometry (i.e., no detailed transition features) to estimate stress concentration factors as a function of a transition notch radius. This Minimal Mesh Density (MMD) approach to estimating stress concentrations is discussed in some detail in Tech Brief 120701F Estimating Stress Concentrations with a Minimal Mesh Density Approach.

The relationship that governs the MMD approach is provided in equation 1. This equation relates the stress concentration factor k_t to the K_I stress intensity factor associated with the stress field as defined in equation 2.

Equation 1:

$$k_t = 1 + 2\frac{K_I}{\sigma\sqrt{\pi\rho}}$$

Equation 2:

$$K_I = F\sigma\sqrt{\pi(a+\rho)}$$

Figure 1 provides the definition of the independent variables used in equations 1 and 2. The variable F is the geometric form factor used in calculation of the K field. The utility of defining the effective crack length per equation 2 is that it allows the probe crack "a" to vanish without equation 1 becoming singular. For a given radius ρ, therefore, an estimated k_t can be obtained from probing a stress field in a transition path.

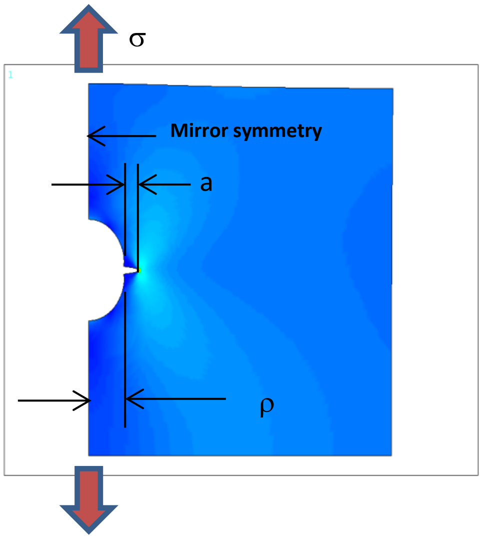


Figure 1 – Half Model of Hole in Infinite Plate

For a given K_I field *the k_t is minimized by maximizing the radius of curvature tangent to the first principal stress.* The maximization of the radius of curvature (ρ) is typically achieved with compound radii or an ellipse. Spatial constraints often create a situation where a portion of the compound transition feature will have a smaller radius of curvature than a simple radius. This portion of the transition feature, however, when it is located away from where the first principal stress is at a maximum, will still result in a lower concentration of stress compared to a simple radius.

Using a crack front to probe the stress field enables the designer to identify where the maximum first principal stress occurs and thus where the location of the maximum radius of curvature should be placed. Three examples of this technique are provided in this brief. The first is the transition of a stepped bar. The second is a T-Head bar supported so as to create a re-entrant corner and the third example is a transition at the heel of a bolted flange to a case.

In each of these examples an ellipse with a minor axis having the same dimension as the simple fillet radius is used in an attempt to reduce the elastic k_t. The design task is to identify the orientation or location in the transition where the elliptical fillet should be located.

Case 1: Stepped Bar

The desired location of the elliptical fillet in the transition of a stepped bar can be determined by inspection. Seeing how the stress intensity probe works with this simple example, however, provides insight on how to use the technique in complex transitions that cannot be assessed by inspection.

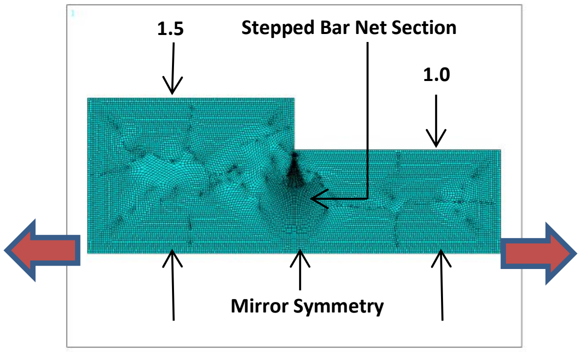


Figure 2 – Stepped Bar Example

The stress field in a simple transition fillet indicates that the location of the maximum first principle stress occurs in the net section side of the transition. This can be seen by the bias of the concentrated fillet stress in Figure 3.

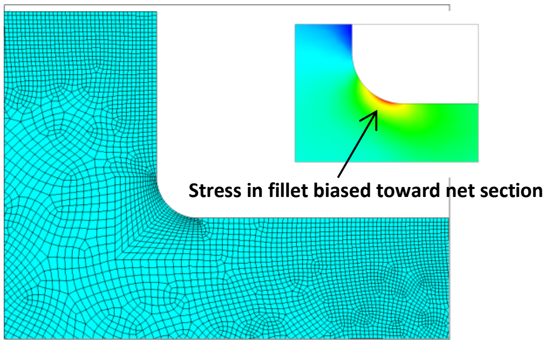


Figure 3 – Concentrated Fillet Stress in Step Bar

Introducing a crack into the de-featured transition, as shown in Figure 4, and then evaluating the K field enables the designer to assess how the maximum radius of curvature should be oriented to minimize the stress concentration. The K field for this crack probe can be found in Table 1. *The K field is dominated by the K_I (opening mode) indicating the crack is fairly perpendicular to the first principal stress field.* The major axis of the elliptical fillet (e.g., the largest radius of curvature), therefore, should be tangent to this stress field to minimize the stress concentration.

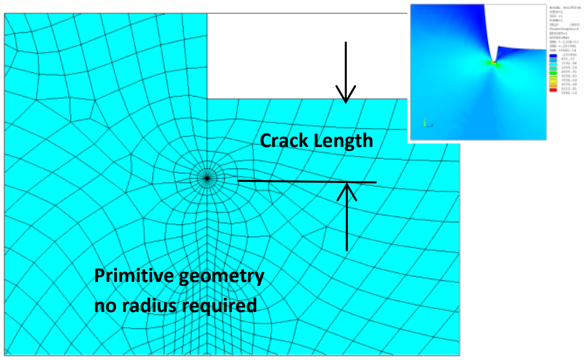


Figure 4 – Probe Crack

**Table 1 – Stress Intensity Values (Plane Stress)
Probe Crack in Figure 4**

Crack Length	K_I	K_{II}	K_{eff}
Inches	ksi-in$^{0.50}$	ksi-in$^{0.50}$	ksi-in$^{0.50}$
0.050	0.66	0.12	0.67
0.075	0.72	0.11	0.73
0.100	0.77	0.10	0.77

With the major axis of the ellipse tangent to the maximum first principal stress field the k_t = 2.16 compared to 2.91 for a 0.10 inch simple radius.

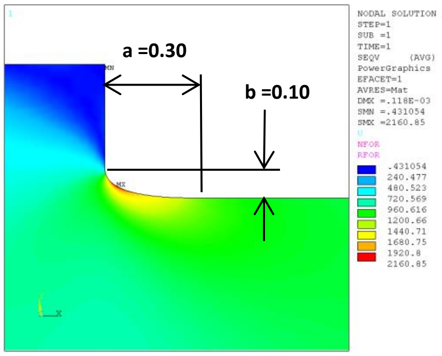


Figure 5 – Properly Oriented Elliptical Fillet

When the probe crack is oriented 90 degrees to the one in Figure 4, the sliding mode K_{II} dominates the K field. Orienting the major axis of the elliptical fillet perpendicular to this probe crack will create a larger stress concentration than a simple radius. The reason for this is that radius of curvature at the tip of ellipse, which is smaller than a simple fillet radius, is now tangent to the first principal stress field.

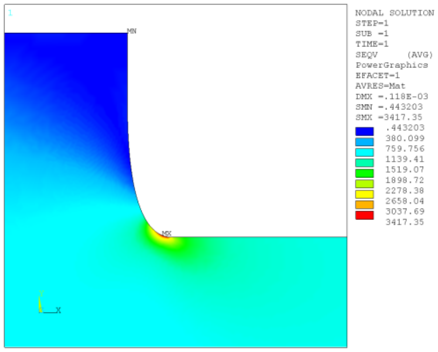


Figure 6 – Improperly Oriented Elliptical Fillet

A k_t of 3.42 is produced with the major axis of the elliptical fillet aligned perpendicular to the maximum first principal stress field.

In summary, the benefit of properly aligning an elliptical fillet in a stepped transition region is significant. A reduction of 26 percent is obtained by properly aligning the elliptical fillet. An improper alignment results in a stress concentration increase of 18 percent over a simple fillet radius.

Table 2 – Stress Concentration Summary Stepped Bar

Transition Feature	K_t
Simple Radius 0.010	2.91
Ellipse Properly Oriented	**2.16**
Ellipse Improperly Oriented	3.42

Case 2: T- Head Bar

The second example, illustrating the use of evaluating the stress intensity field to determine optimal transition geometry, is a T-Head bar. The applied load is equilibrated by a support at the head of the bar. This path creates a re-entrant corner in the bar.

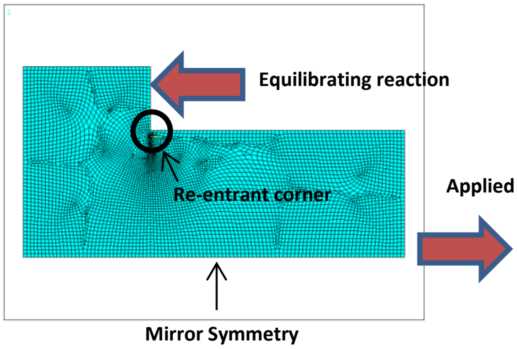


Figure 7 – T-Bar

Orienting the probe crack front in the same direction as in the step bar; shown in Figure 8; provides results indicating the probe is not aligned perpendicular to the first principal stress field. The indicator that the crack probe is mis-aligned is the relatively high value of K_{II} (in-plane shear) relative to the effective K field. The K_{II} is approximately 36 percent of the effective K field. For stress intensity values reference Table 3.

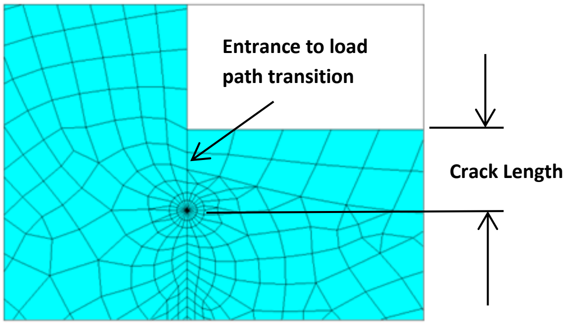


Figure 8 – First Crack Probe for T -Bar

The reason for this change in the K field behavior is that the transition has become a re-entrant corner. This is due to the load flowing around the corner to be equilibrated on the face of the T-Head. The results for this crack probe are summarized in the table below.

Table 3 – Stress Intensity Values (Plane Stress)
Figure 8 Crack Probe

Crack Length	K_I	K_{II}	K_{eff}
Inches	ksi-in$^{0.50}$	ksi-in$^{0.50}$	ksi-in$^{0.50}$
0.050	1.82	0.70	1.95
0.075	1.87	0.73	2.01
0.100	1.91	0.75	2.05

Since the initial probe crack was not well aligned with the first principal stress field two other probe orientations were employed. One was oriented 45 degrees to the initial crack and the other 90 degrees. The probe oriented 90 degrees to the original crack resulted in K_{II} values similar to the values in Table 3. The probe 45 degrees to the original K field evaluation, however, is well aligned with the first principal stress field. The results are provided in Table 4.

In complex load path behavior the technique of employing three probes can be used to identify the direction of the maximum first principal stress field. The resulting K_I values can be curve fit with a second degree polynomial to identify where the maximum radius of curvature should be located in the transition.

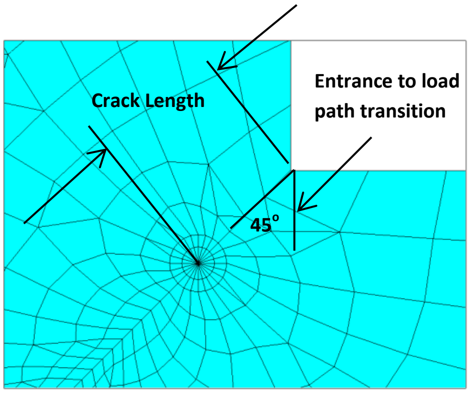


Figure 9 – Crack Probe
Perpendicular to First Principal Stress

Table 4 – Stress Intensity Values (Plane Strain)
Figure 9 Crack Probe

Crack Length	K_I	K_{II}	K_{eff}
Inches	ksi-in$^{0.50}$	ksi-in$^{0.50}$	ksi-in$^{0.50}$
0.050	2.54	0.03	2.54
0.075	2.63	0.10	2.63
0.100	2.73	0.10	2.73

In this particular load path no reduction in concentrated stress is obtained with an elliptical fillet regardless of its orientation. The reason is that the maximum first principle stress field occurs 45 degrees to the entrance of the load path transition. The introduction of an elliptical fillet actually increases the stress concentration since the radius of curvature in the ellipse is at a minimum at this location.

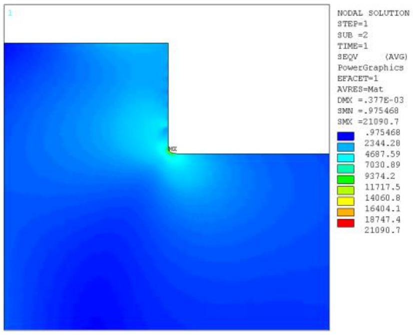


Figure 10 – Major Axis of Ellipse Parallel to Net Section

The elliptical fillet has a major axis of 0.06 inches and a minor axis of 0.02 inches. The resulting elastic stress concentration is 21.1. A simple 0.02 inch radius produces a k_t of 20.0.

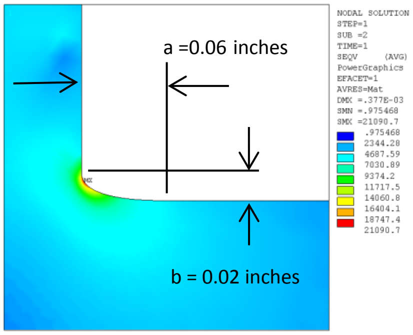


Figure 11 – Ellipse Dimensions

In this load path the maximum first principal stress occurs 45 degrees to the entrance into the transition feature. Maximizing a simple fillet radius provides the largest radius of curvature tangent to the maximum first principal stress field (e.g., perpendicular to probe crack) and hence the minimum k_t. If a fillet radius equal to the major axis of the ellipse is employed in the transition (i.e., 0.06 inches), the resulting k_t is 12.2.

Table 5 – Stress Concentration Summary
T-Bar

Transition Feature	K_t
Simple 0.020 inch Radius	20.0
Ellipse MA Parallel to Net Section	21.1
Ellipse MA Perpendicular to Net Section	21.7
Simple 0.060 inch Radius	12.2

Case 3: Bolted Flange to Case Transition

The last example considered in this brief is a bolted flange. The bolted joint is shown in Figure 12. This case study illustrates the following:

- How load flow through the transition region influences the k_t,
- The use of the crack probe for checking orientation of an elliptical fillet
- The benefits of elliptical fillets in bolt joint transitions
- The use of the Minimum Mesh Density (MMD) technique to estimate the transition k_t using primitive geometry.

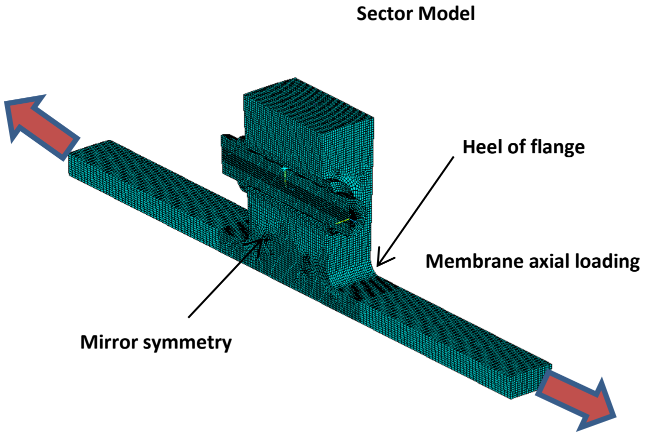


Figure 12 – Bolted Joint Example

Load Flow in the Transition Path

In a bolted joint the stress concentrated at the heel of the flange can be a function of the bolt pre-load. When this occurs the joint is experiencing lift off between the flanges. As the joint is loaded the flanges begin to separate around the region of the bolt hole that initially was in compression. When lift off is present bolt fatigue is typically more limiting than any other feature in the joint. The increase in the stress concentration at the heel transition, however, illustrates how the change in a load path influences the k_t even though the transition geometry has not changed.

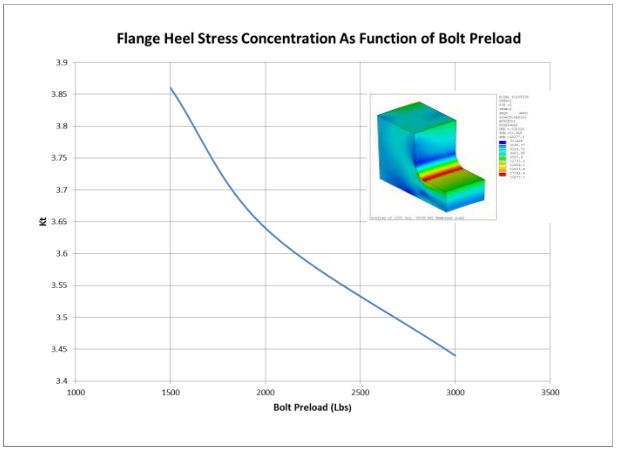
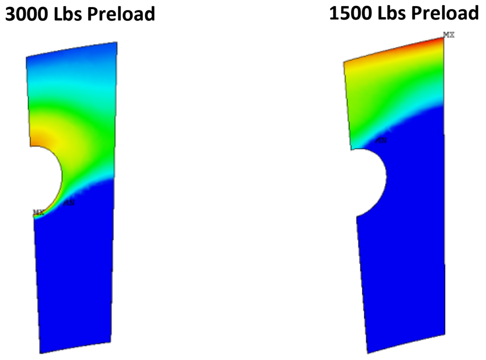


Figure 13 – K_t as Function of Bolt Preload

The change in the load path through the joint can be visualized by seeing the change in contact pressure between the two flanges as a function of bolt preload.


3000 Lbs Preload

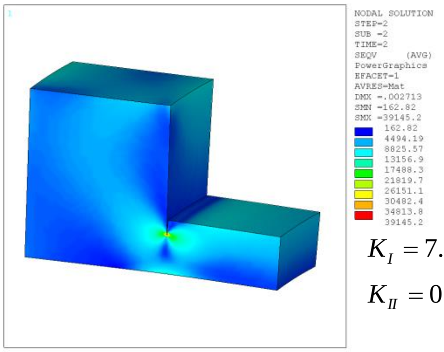
1500 Lbs Preload

Figure 14 – Flange Contact Pattern
Equivalent Membrane Axial Load Applied

As the flanges separate, the effective ratio between the net section (e.g., case membrane) and gross section changes. This is analogous to changing the D/d ratio in a stepped bar.

Checking K Field with Crack Probe

Employing a probe crack to evaluate the behavior of the K field at the heel of the flange indicates that the major axis of an elliptical fillet should be parallel to the case (e.g., perpendicular to the flange). If the elliptical fillet is inverted the stress concentration will be higher than a simple radius.



$$K_I = 7.6ksi\sqrt{in}$$

$$K_{II} = 0.65ksi\sqrt{in}$$

Figure 15 – Probe Crack – 1500 lbs. Bolt Preload

Incorporating an elliptical fillet with a major to minor ratio of 2.6 and with a preload of 1500 lbs. produces a k_t of 2.52. This is a reduction of 35 percent over a simple radius.

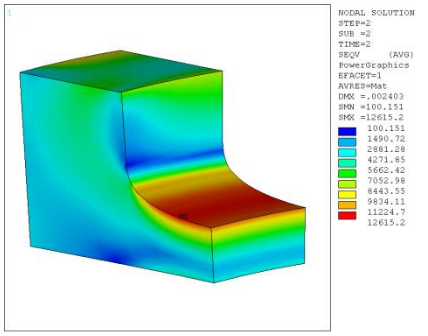


Figure 16 – Stress Field at Flange Heel
Elliptical Fillet (a/b = 2.6)

Additional Benefits of an Elliptical Fillet

Incorporating an elliptical fillet to reduce the stress concentration rather than a larger simple radius provides additional benefits. The elliptical fillet allows the bolt circle to be pulled in closer to the case membrane. This will further minimize the stress concentration at the flange heel as well as minimize the prying on the bolt. Minimizing prying on the bolt by minimizing the offset of the bolt from the line of action of the applied load is one of the most effective means of addressing flange lift off and bolt fatigue. For additional information regarding bolted joint design see Tech Brief 080501F Optimizing Bolted Joint Geometry for Fatigue Loading.

Estimating Stress Concentration with MMD

Lastly, using the Minimal Mesh Density (MMD) approach to evaluate the flange heel k_t for a simple fillet radius of 0.125 inch provides a value that is within 6 percent of the actual value. Once again an outline of this method can be found in Tech Brief 120701F Estimating Stress Concentrations with a Minimal Mesh Density Approach. This approach provides good estimates of stress concentrations using primitive geometry definitions.

Conclusion:

Evaluating the stress intensity field (K field) in load path transition regions with crack probes provides a quantitative and efficient means of guiding a designer towards an optimal detailed design. The primary objective function is to maximize the radius of curvature of the transition feature tangent to the direction of the first principal stress field.

The technique frees the designer from having to have *a priori* knowledge about a complex stress field before attempting to minimize stress concentrations in potentially fatigue limiting areas. It also can be used in transition regions that are defined with only primitive geometry features. As shown in the bolted flange case study, evaluating the K field and employing the MMD method allows a designer to estimate within engineering accuracy the stress concentration of transition features before they are added to the geometry base. Combining this technique with the ease of using ANSYS 14.5 to incorporate crack probes into base geometry provides an efficient design tool for identifying detailed geometry which will minimize the accumulation of fatigue in load path transitions.