Designing Mechanical Systems for Suddenly Applied Loads

Integrated Systems Research
May, 2003

Tech Brief 030501D

Abstract

The design of structural systems primarily involves a decision process dealing with three parameters: geometry, materials and loads. Of these three, load is typically the parameter over which the design engineer has the least control. Understanding how dynamic loads develop in a system, however, is vital in employing geometry and material selections that provide robust design strategies for systems carrying time varying loads.

This tech brief addresses a class of loads typically referred to as suddenly applied. Suddenly applied loads are characterized by rapid changes in magnitude, which can result in structural responses significantly greater than for a load of the same magnitude but gradually applied. Whether or not a change in load magnitude is rapid enough to be classified as suddenly applied is governed by the mass-elastic properties of the system and the rise time of the load. The resulting system response to a time varying load, therefore, is both a function of how rapidly the magnitude of the load changes and parameters that the designer typically has more control over such as geometry and material selection.

Identifying the characteristics of time varying loads is vital in developing an optimal strategy in minimizing the dynamic response of a system. In this brief the characteristics of suddenly applied loads are compared to two other types of dynamic loads: impact and harmonic. A summary of typical strategies employed for handling each type of load is discussed. Additionally, a case study of an industrial tumbler is provided illustrating the behavior of a mechanical system to suddenly applied loads and design strategies employed to address them.

Structural Response to a Suddenly Applied Load:

It can be shown, based on the conversation of energy that the maximum response of a single degree of freedom to a suddenly applied load can be no greater than twice of one that is gradually or statically applied. For systems where multiple degrees of freedom are excited it is possible to obtain responses greater than a factor of two in local regions of the system. It is therefore prudent to carry out the necessary evaluation when participation of multiple modes exists.

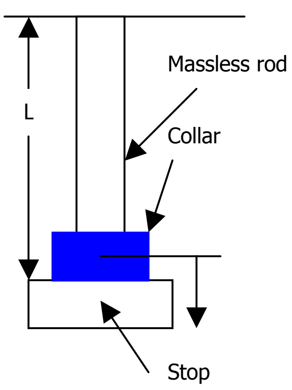

Figure 1

Consider the system shown in Figure 1. A collar with mass M is suspended just above the stop at the end of a massless rod. From the conservation of energy the strain energy in the massless rod will equal the change in potential energy of the collar when it is released.

Based on energy conservation when the collar is suddenly released the maximum deflection response of the rod is twice that of a statically applied load.

For a statically applied load the rod deflection is:[1]

Equation 1.0
$$\delta = \frac{F}{K} = \frac{Mg}{K}$$

[1] K = AE/L is the axial spring rate of the rod and g is the gravitational constant

When the collar is suddenly released, however, the response is :

Equation 2.0
$$0.0 = \Delta U - \Delta PE$$
$$0.0 = \frac{1}{2}K\delta^2 - Mg\delta$$
$$\delta = \frac{2Mg}{K}$$

The complementary energy of a system develops as kinetic energy when a load is suddenly applied. Figure 2 shows the response of a mass-spring system when a load is statically applied. When a load is statically or gradually applied the complementary energy is accounted for by the support reaction forces[2] rather than a mass reaction. Since the displacements are zero at the reactions the work done by these forces and therefore the complementary energy is zero. When a load is suddenly applied the body forces associated with the acceleration of the system's mass makes the complementary energy non-zero.

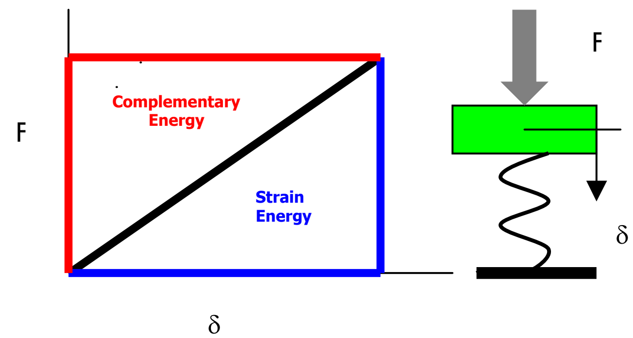

Figure 2

As a designer, identifying how rapidly a load needs to be applied to create a dynamic response is vital in evaluating whether or not the actual load conditions have been accounted for in sizing load path elements. Additionally, identifying the type of dynamic load (suddenly applied or impact) will determine the best design strategy to employ.

Determining When a Load is Suddenly Applied:

A load is suddenly applied when its rise time is less than half the fundamental period of the structural system. For example, if the system has a fundamental mode at 10 Hz then any load that develops in less than 0.050 seconds will create a dynamic response. The amount of overshoot that the system will experience is a function of both the slew rate of the rise and the duration of the applied load. Unlike systems that are harmonically excited near natural frequencies, damping has little influence on the peak transient response.

At first one might infer from this behavior that a designer would always want to maximize the fundamental natural frequency of structural systems in an attempt to avoid the potential of experiencing transient overshoots. Seldom, however, does a system only have to deal with one type of load and therefore maximizing the fundamental frequencies of a structure is not necessarily always the best strategy.

[2] Work is force acting through a distance. Reaction forces exist at the system supports but the displacements are zero and therefore no work is done.

Competing Loading Conditions:

Mechanical systems often need to be designed to handle dynamic loads that have competing characteristics. A reciprocating engine mount system is an example of such a situation. In this case it is very desirable and necessary to have low support system natural frequencies to provide harmonic isolation between the engine and chassis. The low frequencies account for the rigid body mass participation of the engine block that creates high impedance between the engine and chassis in the frequency range where the harmonic excitation of the reciprocating elements exists. In selecting isolation mounts, however, the designer also has to consider the capacity of the mounts to carry loads and limit displacements developed by sudden support displacements such as the automobile hitting a curb or pothole. The stiffness of the mounts are typically governed by the fundamental frequencies required to develop isolation for the harmonic loads. The suddenly applied loads and support displacements, however, typically control the required load capacity of the mounts. The design effort involves selecting mounts that accommodate both competing parameters.

Visualizing the behavior of loads in the frequency domain can oftentimes enhance a designer's understanding of the response characteristics of a system when competing loads are present. Discrete energy bands in the frequency domain characterize harmonic loading. Isolation strategies and tune dampers lend themselves very well to this type of loading because both of these strategies allow the designer to locate the mass participation in frequency bands where input energy is not present. Suddenly applied loads and impact loading, however, are broadband in the frequency domain and oftentimes tend to be concentrated in the lower frequency bands. This characteristic makes it difficult to avoid significant mass participation when low frequency modes are present.

Frequency Domain Characteristics of Dynamic Loads:

Figure 3 provides an example of the frequency content of a periodic trapezoidal pulse and a steady state sinusoidal load. This example illustrates the inverse relationship that the time and frequency domains have with each other. A steady state sinusoidal load in the time domain is broadband while in the frequency domain it is a single spectral line at the frequency of the load. A periodic pulse, on the other hand, is relatively discrete in the time domain but results in an energy distribution that is very broadband in the frequency domain.

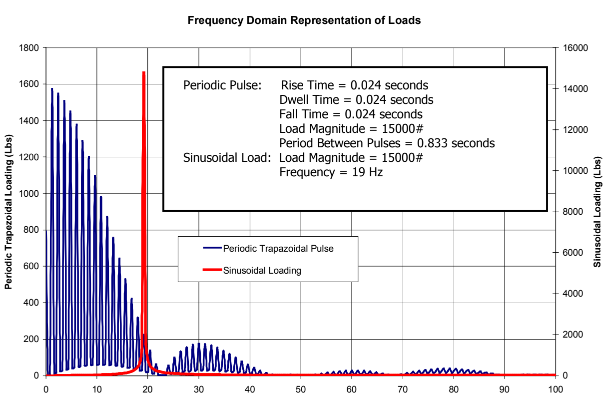

Figure 3

The response of a mechanical system to dynamic loading is the product of its frequency response function with the imposed loading function in the frequency domain. The time domain response for the same system is the convolution of the system's unit impulse response function with the load time history. This illustrates the principle that multiplication in the frequency domain is equivalent to convolution in the time domain. Since computationally multiplication is significantly faster than convolution, many real time control processes are performed in the frequency domain to take advantage of this efficiency.

Figure 4 provides the frequency response function (FRF) of a 10 Hz single degree of freedom (SDOF) system. The system has a significant amount of damping with a beta value of 0.0064 being employed. Notice that there are three distinct regimes of the FRF. The first regime is the response of the system from a static condition to approximately 80 percent of its natural frequency. The regime is dominated by the real response, which is in phase with the applied load and has little amplification relative to a static response.

The second regime is in the neighborhood of the system's natural frequency. Here the imaginary response dominates. The imaginary response is 90 degrees out-of-phase to the applied load. The only parameter that limits the response of the system at its natural frequency is damping. Damping dominates at a system's resonance. This is the reason why damping doesn't play a large part in the peak response of a system to suddenly applied loads. Suddenly applied loads have energy distributed over a broad frequency band. The response, therefore, is governed by the mass-elastic participation of the system and not just the damping that occurs at its natural frequency. Although damping does not significantly influence the system's peak response, the rate of decay of the response is governed by it.

The third regime occurs above the system's natural frequency. In this frequency band the response is once again primarily real but is now 180 degrees out-of-phase with the input. Additionally, the response is lower than the system's static response, which is the basis for creating harmonic isolation.

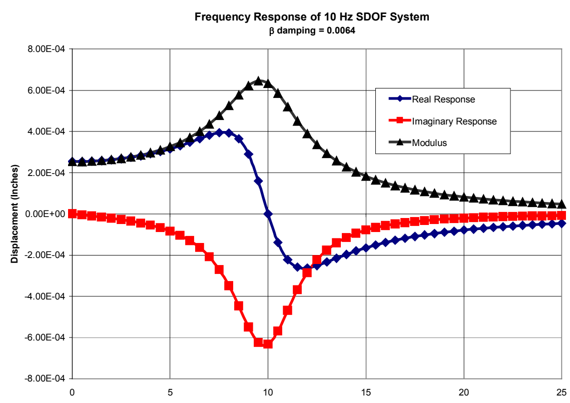

Figure 4

On the next page, Figure 5 provides the time domain response of the 10 Hz SDOF system for both the periodic pulse and sinusoidal forcing functions shown in the frequency domain in Figure 3. Both forcing functions have a peak load of 15,000 pounds. The responses, however, are significantly different with the periodic pulse creating a response five times greater than the sinusoidal forcing function. Once the system's FRF, in Figure 4, is overlaid with the frequency domain load distribution in Figure 3 it becomes very apparent why this occurs. The energy associated with the periodic pulse is concentrated in the frequency range where the system has its maximum response while the sinusoidal input is present only in the regime where the system's response is attenuated. A good design strategy will take advantage of both the system's response characteristics and how the excitation energy is distributed in the frequency domain.

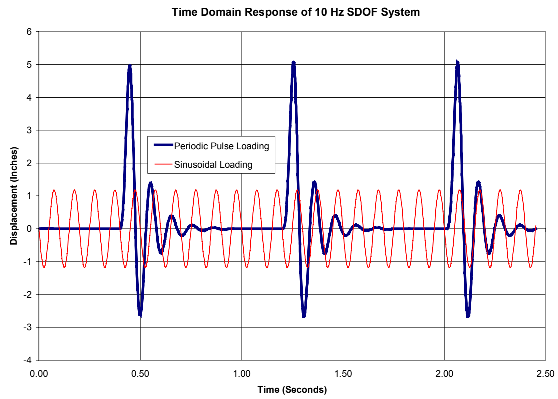

Figure 5

Impact Loading:

Momentum transfer is the characteristic that distinguishes an impact load from one that is suddenly applied. As shown earlier the maximum response to a suddenly applied load can be no greater than a factor of two with respect to an equivalent statically applied load for a SDOF system. It can also be shown from equation 2.0 that this dynamic response is independent of the system's stiffness. This is not the case, however, when there is a transfer of momentum present in the system. When momentum transfer occurs, minimizing the stiffness of the system is the typical strategy employed to minimize the dynamic loads developed during the event.

Figure 6, provides the stress response in a massless rod when a mass weighing 100 pounds is dropped from a height 10 inches above the stop. The stress response surface is a function of the length and Young's modulus of the rod. The surface is a rising ridge and indicates that the minimum stress occurs as the rod's spring rate approaches a minimum.[3] This illustrates the principle that the impulse equals the exchange of momentum in the system. The longer period of time over which the momentum exchange occurs the lower the impact loads will be. Lowering the system's stiffness to minimize impact loads tends to be antithetical to the approach typically taken to minimize the response to suddenly applied loads. This makes it important for the designer to correctly identify when a load is suddenly applied or when momentum transfer is present. Often the difference is readily apparent. The industrial tumbler case study, however, is an example where the dominant type of drive train loading wasn't readily appreciated and proposed modifications to solve the resulting dynamic loading problems would have actually exacerbated them.

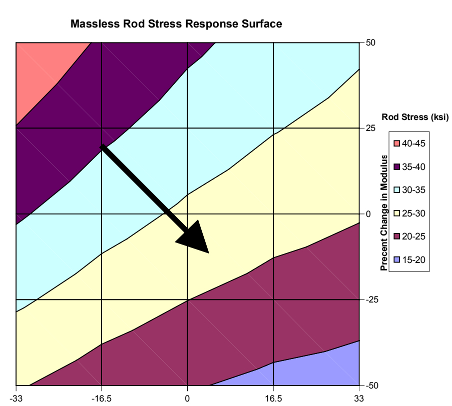

Figure 6

[3] Note that increasing the diameter of the rod will also lower the rod stress. The impact load, however, will increase but not as quickly as the rod's cross-section.

Case Study: Industrial Tumbler

A large industrial tumbler experienced significant scoring of the drive pinion and excessive vibration levels of the power drive platform, which created periodic production shutdowns. The manufacturer of the tumbler had a significant number of units, of the same design, in production that had provided satisfactory vibratory and gear life performance. These units had been in service for five to ten years.

Upon investigation of the problem unit it became clear that the fundamental mechanism creating the observed distress was due to discontinuities in the girth gear, which was made in 30-degree cast segments. These discontinuities created a periodic pulse that was amplified by the response of the drive platform. The major difference between the units that had successfully operated for five to ten years and this particular application was the modal participation of the drive platform. The successful applications had drive platforms with fundamental lateral frequencies 3 to 4 times the frequency of the unit experiencing the distress.

The nature of the problem was essentially displacement controlled. As the split line of a girth gear section entered the pinion-loading zone an enforced displacement between the pinion and gear would occur. In the applications where the lateral frequencies of the drive platforms were high the vast majority of the displacement was carried by the girth gear and shell and the relative gear teeth sliding in the loading zone was essentially equal to the slight misalignment between girth gear sections.

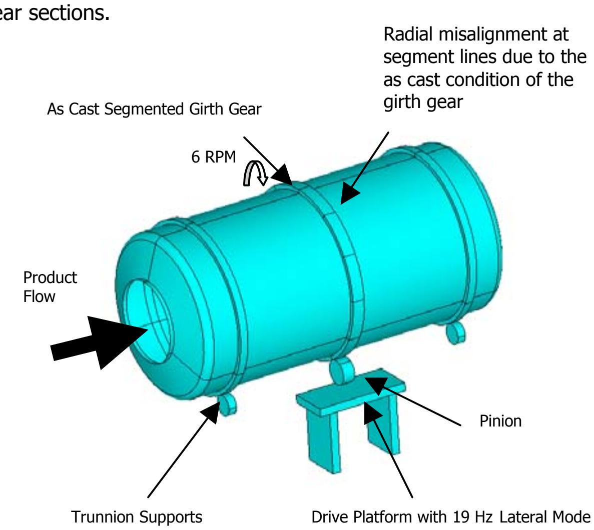

Figure 7

In the application that had the low frequency lateral platform mode, however, the response to the girth gear section misalignment was significantly amplified. The lateral natural frequency of the platform was approximately 19 Hz, which corresponded to a period of 0.052 seconds. The load created by the girth section misalignment passing through the loading zone developed in approximately 0.023 seconds and was sustained another 0.045 second before exiting the engagement. The rapid load rise relative to the platform's modal participation created a dynamic response of the drive platform that significantly increased the relative sliding between the gear and pinion as shown in Figure 8. This made it virtually impossible to sustain any hydrodynamic film in the contact zone and generated vibration levels in the drive platform that periodically shutdown production.

The obvious solution to the problem was either to replace the drive platform with one having modal characteristics similar to the other units that were in operation or replace the as cast girth gear with a design that would eliminate the periodic forcing function created by the girth gear segment misalignments. Neither one of these solutions were very attractive from a cost standpoint. The cost of the obvious solution(s) drove the team working on the problem to attempt to develop other less costly alternatives. In the end, however, the physics of the problem prevailed and the solution was obtained by replacing the girth gear. Reviewing why these "cost effective" alternatives[4] would have failed can help develop a greater appreciation for handling suddenly applied loads.

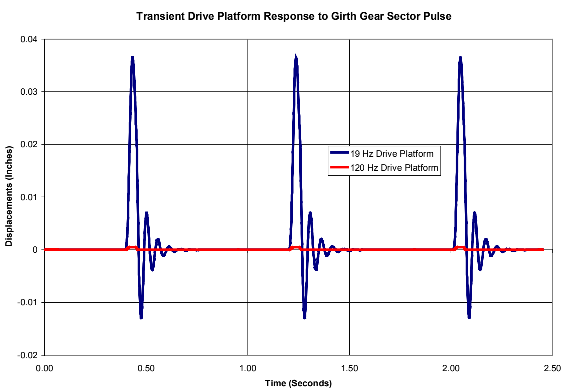

Figure 8

The two suggested alternatives were either to tune damp the platform or soft mount the pinion to isolate it from the platform. Both approaches were suggested due to a lack of appreciation of the broadband nature of the loading in the frequency domain and looking at the problem as an impact rather than a suddenly applied load.

Isolation Mount Alternative:

This suggested alternative neglected several key aspects of the functionality and loading characteristics of the girth gear sectors. First the nature of the loading is a pulse in the time domain, which concentrates the vast majority of the energy in a frequency band below the frequency of the pulse period. In this case the vast majority of the pulse energy was below 15 Hz. Attempting to isolate the pinion from the drive platform with a soft mount arrangement would only create a local pinion mode in the frequency range where most of the excitation energy existed.

Another line of reasoning used to advance this proposal was to assert that soft mounting the pinion would reduce the magnitude of the applied sector load because it would decrease the impulse load in the drive systems. This was based on the assumption that the load was primarily created due to a transfer of momentum between the girth gear and pinion systems. If this had actually been the case decreasing the mount stiffness would have little influence on the time over which the girth gear and pinion would make contact. In an impact scenario the time over which the end of the girth gear segments made contact with the pinion would have had to increase in order to decrease the impulse load.[5] Since the stiffness of the pinion mount doesn't influence the period of time that girth and pinion are in contact a decrease in dynamic load would not be expected even if the loading mechanism had been due to momentum transfer.

The actual loading mechanism between the girth gear imperfections and pinion, however, was primarily displacement controlled. The girth gear and tumbler shell acted as springs in series with the pinion and drive platform. The enforced displacement created by the girth segment imperfections would be distributed between the girth gear and pinion based on their relative stiffness. As with any series system, the element with the lowest stiffness will have the greatest amount of displacement. If the pinion mounts were soft enough, the argument went, then the pinion would carry all of the displacement and the pulse load would be virtually non-existent since the spring rate of the mounts would be so low. The problem with this approach, however, was that now the steady state loads would create deflections between the gear and pinion so large that they couldn't function properly. Soft mounting the pinion was not a solution.

This line of reasoning, however, underscores the importance of appreciating the competing characteristics between various types of dynamic and steady state loads that are present in a system at the same time. Addressing one type of loading condition without consideration of the other types of loads present in the system can oftentimes lead to "solutions" that will actually exacerbate the problem.

Tune Damping of the Drive Platform:

The second suggestion was to tune damp the modal participation of the drive platform. A tune damper distributes the mass participation of a single mode between two. When the tune damper is added two modes are created with one mode having a higher frequency and the other a lower frequency than the original SDOF system. The frequency range between the modes tends to have high impedance. The mass line of the lower mode is 180 out-of-phase with the stiffness line of the higher mode resulting in a condition often referred to as anti-resonance.

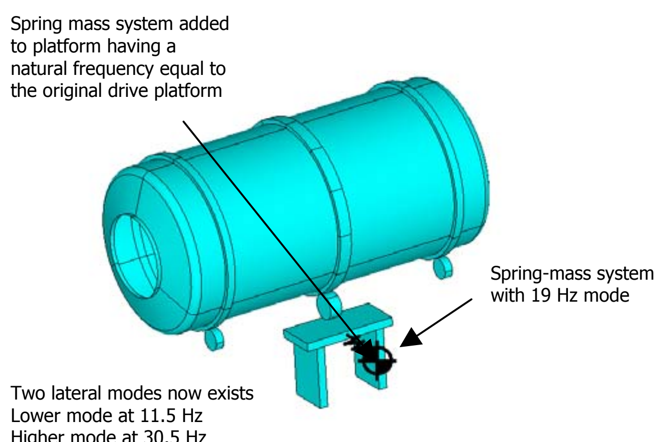

Figure 9

Again this approach overlooked the broadband nature of the pulse loading in the frequency domain. The amount of energy concentrated in the anti-resonance region between the modes was relatively small compared to amount that would excite the lower mode. Hence, the response attenuation would be minimal. If the problem had been due to the loading at the gear passing frequency then this approach would have had merit. In that case the energy would be concentrated in a narrow frequency band where the anti-resonance exists.

In order to develop a high impedance frequency range a mass-elastic damper has to be added to the system that has its own resonance at the frequency of the system to be damped. In addition, the damper has to be attached in a region of significant mass participation in the original mode. As shown in Figure 10, on page 5, the addition of the damper creates a significant attenuation of the response in the frequency range of 19 Hz where the original system resonated. The response to the pulse loading, however, is not reduced due to the relatively large amount of input energy below 15 Hz and the participation of the 11.5 Hz mode.

[4] Obviously, a cost effective failure is an oxymoron.
[5] Impulse is the product of force and time. The product of the two will equal the transfer of momentum in an impact event. The two parameters, force and time are therefore inversely related to each other.

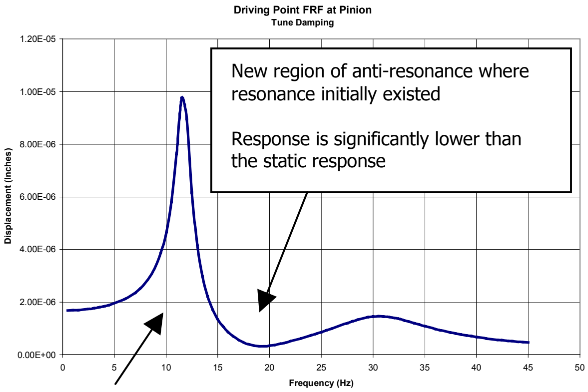

Figure 10

As shown in Figure 11, not only does the tuned damper not attenuate the peak response but the range of the response is also increased. This is due to the pulse width being longer with respect to the period of oscillation of the lower mode, which is now 11.5 Hz.

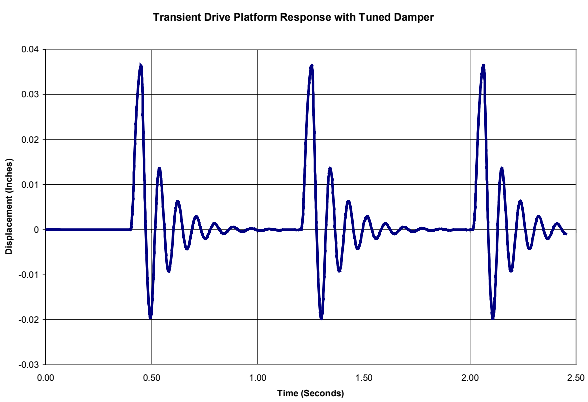

Figure 11

Summary:

In general the distribution of a load's frequency content is the principal feature by feature by which dynamic loads can be categorized. Under the class of broadband frequency loads two primary types exists. The first are suddenly applied loads. The rise time of these loads are less than half the fundamental period of the system. The resulting body forces in the system cause the complementary energy in the system to exhibit itself as kinetic energy and produce a dynamic response. Forces created by the exchange of momentum are the second classification of broadband loads. Since these loads have energy distributed across broad frequency ranges isolation and tuned damping strategies tend not to be the most efficient means of addressing these loads.

Harmonic loads have energy concentrated in narrow frequency bands and therefore lend them to isolation and tuned damping strategies. It is very common, however, to have both transient, harmonic and steady state static loads all present in the same system creating a need for competing design strategies.

Some things to keep in mind when dealing with dynamically loaded systems are to first identify the primary type of loading present in the system. Is the energy distribution broadband or narrowly concentrated in terms of its frequency content? Determine if the generated forces are displacement or load controlled. Determine if any momentum exchange is present. The characteristic of the primary system load will tend to dictate the overall design strategy for the system.

Secondly, determine if there are any means by which one or more of the competing loads can be eliminated. Oftentimes this cannot be done, but as in the case of the industrial tumbler this was the solution employed to address the performance problems.

Thirdly, look for means of addressing secondary loading events without influencing the strategy for the primary loading mechanism in the system. A typical example of this is the use of snubbers in soft mounted reciprocation systems. The soft mount provides the isolation for the harmonic loading but the snubbers limit the travel of the equipment when transient loads are present thus preventing an overload condition of the soft mounts.

Lastly, develop an understanding of the receptivity of the system in areas where loads are being transferred into the system. Generating frequency response functions of the system at these interfaces provides a measure of how responsive significant mass elements within the system will be due to energy transferred at these points. Combining these FRF and an understanding of the frequency content of the load mission can help a designer identify how best to utilize the two main design parameters that he or she have at their disposal: material selection and geometry.

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