Estimating Notch Strains with Net Section Plasticity

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Abstract

This tech brief provides an example of estimating strains in a local notch root when net section plasticity is present. The example employs both a modified Neuber and a Strain Energy Density model to predict the local strain in the notch. The SED model is based on work done by Ellyin and Kujawski.[1]

The brief illustrates the practical importance of considering the level of net section stresses as well as the concentrated stress in estimating local strain behavior. The plasticity in ductile material below the 0.2% yield value can have a significant influence on the local notch strain and predicted life.

Material Behavior

The cyclic behavior of a material needs to be considered when selecting an alloy to be employed in a fatigue situation. The alloy's cyclic stress-strain behavior can vary significantly from its monotonic behavior.

Alloys can experience strain hardening, softening or a mix mode behavior when cycled. Strain hardening occurs when the stress increases for a given strain level. The opposite behavior occurs for a strain softening material and a mix mode alloy experiences softening over a lower range of strain and then hardening as the strain level increases.

Figure 1 provides the cyclic stress strain behavior of the low carbon steel A36.

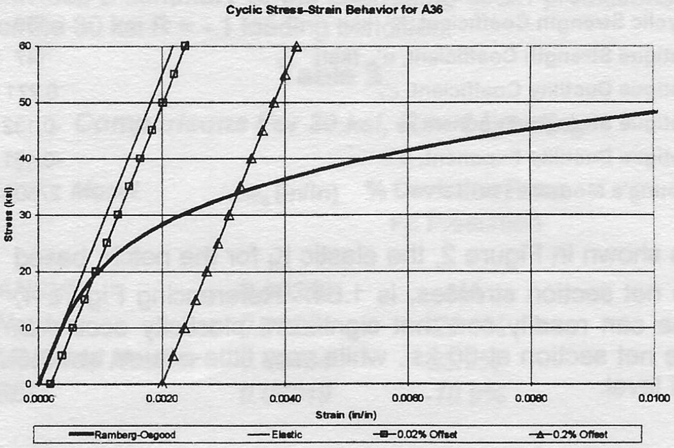

Figure 1

Several important features of ductile materials can be pointed out from the Ramberg-Osgood representation of the A36 stress strain curve. The first feature is that a significant amount of plasticity occurs well below the 0.2% yield value. The strain range between 0.02% and 0.2% is often used to generate the plastic term in the Ramberg-Osgood material model.[2] For this alloy, 0.02% yield is approximately 20 ksi, which is considered the onset of plastic behavior. This is well below the 33 ksi for the 0.2% yield.

The second feature of cyclically softening or mixed behavior alloys, is the onset of plasticity, tends to occur in the neighborhood of one half of the monotonic yield value. Notice that the 0.02% yield value is 20 ksi and the departure of the σ-ε curve from a liner response appears to occur in the neighborhood of 13 to 14 ksi. The average of these values approach one half of the 0.2% monotonic yield value of this alloy.[3] For this reason, manufacturers of cyclically loaded equipment often limit net section stresses to one half the monotonic 0.2% yield. Allowing net section stress above this value produces some measure of gross plasticity in the net section, creating greater local strains in notch features.

When evaluating the robustness of a life prediction, it is important that the design engineer understands both the cyclic behavior of the alloy he or she is working with and the amount of net section plasticity present. The example of a symmetrically notched bar, provided in this bulletin, illustrates the influence of net section plasticity on life predictions as well as providing a comparison between difference plasticity models.

Review of Neuber's Rule

The most common model employed in estimating the local plastic behavior at a notch root is Neuber's rule. The model is often expressed as shown below:

$$K_t^2 \Delta S \Delta e = \Delta\sigma\Delta\varepsilon$$

Equation 1

The left side of the equation represents the product of the peak elastic stress and strain. The right side of the equation is the true stress and strain. The estimates are obtained by substituting the Ramberg-Osgood[4] σ-ε relationship in Δε on the right side of the equation and solving for Δσ. Once Δσ is found, Δε is determined by back substitution into the Ramberg-Osgood relationship.

[1] F.Ellyin,D. Kujawski, *Notch Root Stress/Strain Prediction for Elastic-Plastic Loading*, Res Mechanica 20 (1987) 177-190

[2] Reference MIL-HDBK-5G, 9-68

[3] Using 36 ksi as the 0.2% yield value.

[4] Reference Equation 1, ISR Tech Brief 981201F for the Ramberg-Osgood stress-strain relationship.

For subsequent range pairs, the true stress and strain predictions can be found using the same approach but employing Massing's theory. Massing's theory provides predictions, which are in good agreement with material exhibiting kinematic hardening behavior.[5]

Neuber's Model Modified for Net Section Plasticity:

A modification to this approach is provided in the ISR Tech Brief 981201F. In this approach, the Ramberg-Osgood relationship is used in the Δe term in Neuber's rule as well as for the true strain range $\Delta\varepsilon$. Since the Ramberg-Osgood relationship includes both the elastic and plastic terms, if there is gross plastic behavior present in the net section this approach will account for it. Employing this method ensures plasticity in the net section is always accounted for. If little net section plasticity is present, the predictions will trend towards those provided by the unmodified approach.

Strain Energy Density Model:

It has been shown that Neuber's rule, *modified for net section plasticity*, tends to over estimate the notch root strain behavior. To address this situation, strain energy density methods have been developed aimed at improving strain predictions in local notch features.

One such SED model developed by Molski and Glinka is given below:

$$W_n K_t^2 = W_a^{max}$$

Equation 2[6]

Where W_n is the nominal elastic strain energy density and W_a^{max} is the actual maximum strain energy density in the root of the notch.

Kujawski and Ellyin developed another SED model. Using Rice's J – integral definition, a similarity measure of the strain energy density along the surface of the notch was obtained. Based on this similarity condition, the following relationship between the maximum stress and strains to the nominal ones was developed.

$$K_t^2\left(\frac{\Delta S_n^2}{2E}\right) + \frac{1}{1+n'}\Delta S_n \Delta e_n^p = \frac{\Delta\sigma^2}{2E} + \frac{1}{1+n'}\Delta\sigma\Delta\varepsilon^p$$

Equation 3[7]

A comparison is provided between finite element results and the predictions provided by the Neuber and Kujawski/Ellyin models. The finite element results are for a plane stress bar with a semi-circular notch.

Elastic Finite Element Results of Notched Bar:

Figure 2 provides a plot of the elastic stress predictions at the bottom of a semi-circular notch, which has a net section stress of 30 ksi.

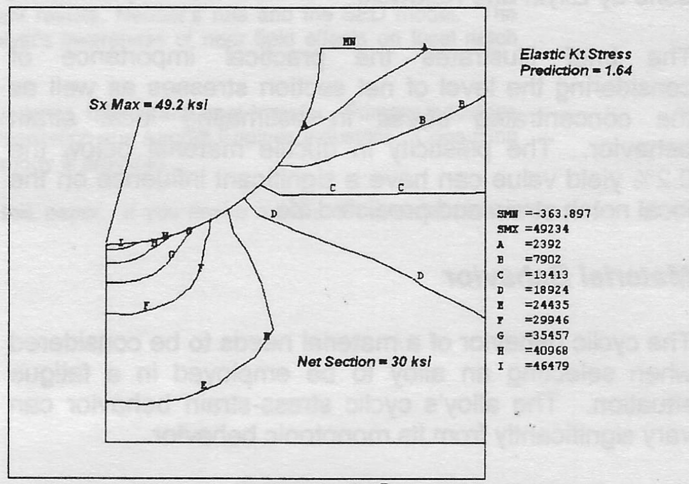

Figure 2[8]

The material used in the analysis is A36. The cyclic properties used in the subsequent plasticity analyses, are listed in Table 1.

Table 1[9]

Material Properties	A36-BM
Cyclic Yield Strength, 0.2% Offset, (ksi)	33.6
Cyclic Strain Hardening Exponent, n'	0.249
Cyclic Strength Coefficient, K' (ksi)	159
Fatigue Strength Coefficient, σ'_f, (ksi)	147
Fatigue Ductility Coefficient, ε'_f	0.271
Fatigue Strength Exponent, b	-0.132
Fatigue Ductility Exponent, c	-0.451
Young's Modulus (ksi)	27500

As shown in Figure 2, the elastic K_t for the notch, based on net section stresses, is 1.64. Referencing Figure 1, one can readily see that significant plasticity occurs in the net section at 30 ksi, while very little occurs at a 15 ksi level.

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[5] Ibid. Equation 3, under the section entitled *Estimate True Stress/Strain Fillet Behavior.*

[6] F.Ellyin,D. Kujawski, *Notch Root Stress/Strain Prediction for Elastic-Plastic Loading*, Res Mechanica 20 (1987), page 179

[7] Ibid., page 180

[8] Analysis performed using ANSYS 5.5.2

[9] University of Illinois, FCP 22, Strain Controlled Fatigue Behavior of Weld Metal and Heat-Affected Base Metal in A36 and A514 Steel Welds, Higashida and Lawrence

The following finite element analyses evaluated the plastic notch behavior for both the 30 and 15 ksi loading conditions.

Plasticity Predictions for Semi-Circular Notched Bar:

The plasticity analyses performed in ANSYS employed a multi-linear kinematic hardening model[10] with an alternating load condition of R = -1. Figure 3 provides the strain predicted at the notch for a net section stress of 30 ksi.

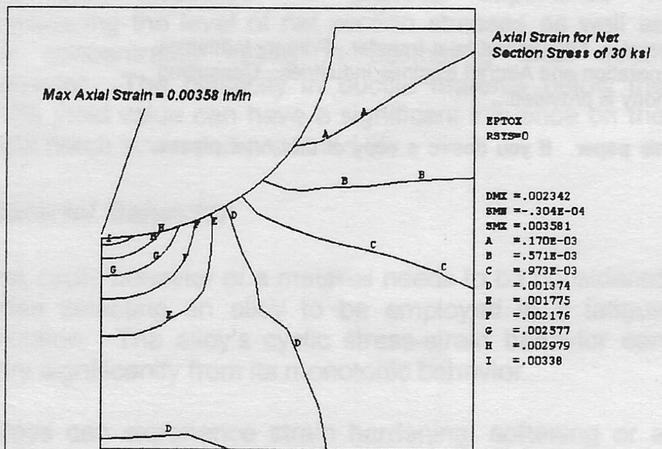

Figure 3

The alternating strain for the R = -1 net section loading of 30 ksi is 0.00358 in/in. The alternating strain is 0.00098 in/in, for R = -1 loading of 15 ksi.

Alternating Strain Predictions:

Based on the various models employed, Table 2, provides a summary of the alternating strain predictions for the 30 ksi R = - 1 loading condition.

Table 2

Comparisons For 30 ksi, R = -1 Loading

Model	$\Delta\varepsilon_a$ (in/in)	% Deviation From FE Prediction
ANSYS	0.00358	NA
Neuber	0.00251	-29.9%
Modified Neuber	0.00484	35.2%
SED	0.00319	-10.9%

Table 3, provides the summary for the 15 ksi, R = -1 loading condition.

Table 3

Comparisons For 15 ksi, R = -1 Loading

Model	$\Delta\varepsilon_a$ (in/in)	% Deviation From FE Prediction
ANSYS	0.00098	NA
Neuber	0.00105	7.4%
Modified Neuber	0.00115	16.7%
SED	0.00101	2.7%

Life Predictions:

Table 4 provides a summary of *average* life predictions based on the plasticity models employed.

Table 4

Life Predictions Based on Manson-Coffin Equation

Model	Cycles to Failure 30 ksi Net Section Alternating Stress	Cycles to Failure 15 ksi Net Section Alternating Stress
ANSYS	21000	2200000
Neuber	65000	1625000
Modified Neuber	8500	1125000
SED	28750	1950000

Summary/Conclusions:

The life summary underscores the importance of accounting for net section plasticity when estimating local notch strain behavior. As seen in the 30 ksi net section loading scenario, employing an unmodified Neuber's model can result in an optimistic life prediction.

When using a modified Neuber's or SED model, it's apparent that the concept of a K_t value is still important even in the age of finite element analysis. The K_t value allows the stress field to be separated into two domains (local and near field) so that the influence of each can be accounted for. Without a K_t value, in the non-finite element plasticity models, these influences cannot be taken into account. For large complex stress histories, using tools such as Neuber's rule or SED models is still much more efficient than non-linear finite element methods. Employing these models also provides the added benefit of being able to cross check plasticity predictions of finite element results.

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[10] Linear to 15 ksi then follows Ramberg-Osgood relationship