Estimating Orthogonal Cutting Tool Forces

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Abstract:

This Tech Brief discusses an approach for estimating cutting tool forces for use in finite element evaluations of work piece deflections. Cutting tool forces vary considerably based on the alloy, cutting tool geometry, material removal rate and cutting speed. An approach for using an energy conservation method in estimating cutting forces is outlined in this paper. Additionally, a means of cross checking the estimates is discussed and illustrated. The cross check employs a Single Shear Plane (SSP) model and is used in a Design of Experiments (DOE) to identify corner points of the tool force design space. The corner points can then be used in the finite element evaluation of the work piece deflection to assess the limits of the fixture design.

Background:

The most straight forward approach to estimating cutting forces is to employ an energy method. Specific energy values associated with machining metallic alloys are readily available. Based on the rate of material removal and the specific energy associated with the alloy being machined, the cutting forces can be estimated based on the conservation of energy.

Due to the actual complexity of the metal removal mechanisms during machining, however, specific energy values vary over a considerable range for the same alloy. The relationship between the shear angle, rake angle, material strain rate hardening and friction are all variables influencing the actual cutting forces.

Solutions to the cutting force problem employ the principle of maximum work. This is based on the deformation caused by the applied stresses resulting in a maximum dissipation of energy. When employing stress values compatible with static equilibrium and yield conditions, however, a state of minimum energy is computed. This results in the estimated cutting forces being either equal or less than that of the actual system (e.g., a lower bound solution). These solutions are obtained from Single Shear Plane (SSP) models.

Upper bound solutions are based on the strain rate of a fully plastic volume rather than stress equilibrium. Obviously, the material is being removed (i.e., in motion) and therefore treating the material removal as a static phenomenon is a step removed from the reality of the actual mechanism. The stresses deduced from the kinematic conditions associated with the deformation of the metal removal will be equal or greater than those that actually occur. These estimates would provide an upper bound. Shear Zone models are employed for these estimates.

Single Shear Plane Models:

A SSP model used to check the reasonableness of an energy conservation calculation assumes rigid body equilibrium of the chip. This calculation tends to provide loads equal to or less than the actual system forces. The most popular lower bound solution is typically referred to as the Merchant model and is the simplest of the cutting force calculations.

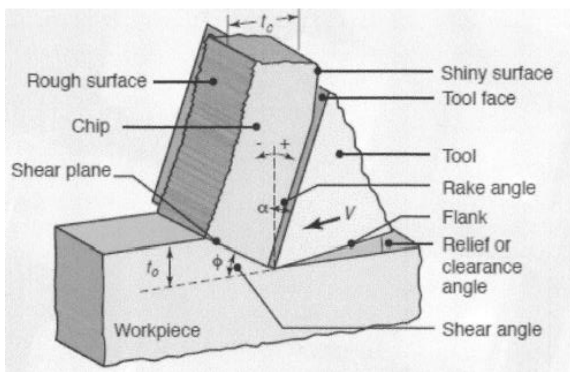

Figure 1 – Single Shear Plane Model

Shear Zone Models:

Shear Zone models attempt to account for the velocity and strain of the material in the plastic zone. The first model attempting to capture this mechanism was based on work done by Oxley. These models can be quite complex and the strain rate and temperature data are oftentimes not readily available.

Due to the difficulty in estimating actual cutting forces, it is prudent to consider evaluating work piece deflection over a reasonable range of cutting forces associated with the alloy and tool geometry. This brief provides a simple means of identifying an expected force level for orthogonal cutting operations and cross checking them with a Single Shear Plane (SSP) model.

The force values from a SSP model can be used as a baseline point for analysis with the realization that the loads could be a lower bound. It is always recommended that when evaluating the deflection behavior of the work piece and clamping capacity of the fixture a response surface analysis of the tool force design space be undertaken to identify the corner points for use in the analysis of the system.

Conservation of Energy Approach:

Table 1 provides estimated specific energy values for machining various types of alloy. The values represent the power consumed at the drive motor. The drive efficiency accounted for in the data is 80 percent. One of the most valuable aspects of this data is that it allows for a relative comparison between alloys. This enables a manufacturing engineer to anticipate the challenge of holding tolerances when an alloy change is specified for a given work piece.

Table 1- Specific Energy[1]

Approximate Specific-Energy Machining Requirements	
Materials	**hp-min/in^3**
Aluminum alloys	0.15-0.40
Cast Iron	0.06-2.0
Copper alloys	0.50-1.20
High-Temperature alloys	1.20-3.10
Magnesium alloys	0.15-0.20
Nickel alloys	1.80-2.50
Refractory alloys	1.10-3.50
Stainless steels	1.10-1.90
Carbon steels	1.0-3.40
Titanium alloys	1.10-1.15

Figure 2 provides a force diagram of the loads acting on the work piece for an orthogonal cutting operation. Fc is the load acting on the work piece in the direction of the cutting tool. It is this force that actually does the machining work on the part.

The separating force Ft is a function of the rake and friction angle. Since virtually no motion occurs normal to the part being machined and therefore no work is done by this force. This force cannot be directly estimated from a conservation of energy approach.

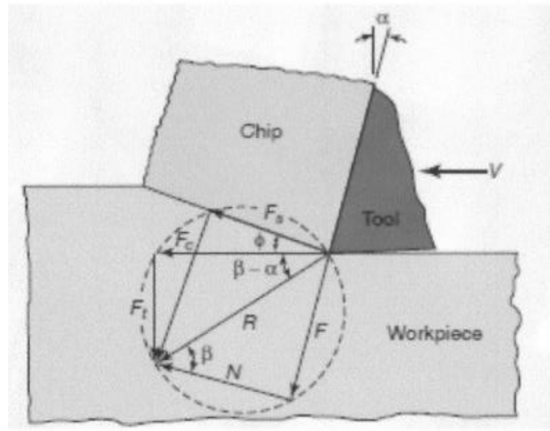

Figure 2 – Force Diagram for Single Shear Plane

To compute Fc using specific energy values requires knowing the cutting velocity and volume of material removal per pass. For example, machining annealed 4130 steel with a orthogonal cutting tool having a rake angle of 25^o, a width of 0.475 and depth of cut of 0.0025 inches at a cutting velocity of 90 feet per minute results in a cutting force Fc = 380 lbs.[2] The separating force Ft is 224 lbs.

Taking the lower bound of the specific energy values for carbon steel (annealed 4130) and the rate of material removal given above results an estimated cutting force of 367 lbs. is obtained using the principle of energy conservation.

The rate of material removal:

$$MRR = 0.0025 \times 0.475 \times 90 \times 12 = 1.28 \frac{in^3}{\min}$$

The power consumed in the cutting process is then calculated:

$$Power = 1 \frac{Hp - \min}{in^3} \times 1.28 \frac{in^3}{\min} \times 0.80 = 1.026 Hp$$

[1] Table 8.3 Manufacturing Processes for Engineering Materials, 5th ed. Kalpakjian and Schmid

[2] Ibid., Table 8.1

Horsepower is converted to Ft-Lbs. per second

$$Power = 1.026Hp \times 550 = 564.3\frac{Ft - Lbs}{\sec}.$$

Power is the product of force and velocity. Given the cutting velocity of 90 feet per minute or 1.5 feet per second, the cutting force Fc can then be computed.

$$Fc = \frac{564.3\frac{Ft - Lbs}{\sec}}{1.5\frac{Ft}{\sec}}. = 376.2Lbs.$$

This estimate is in excellent agreement with the actual cutting data.

In order to estimate the thrust or separating force a SSP model is required. This model can also be used to cross check the cutting force estimation obtained from energy conservation.

Cross Check with Single Shear Plane Model:

The Single Shear Plane model employs the force diagram shown in Figure 2. The relationship between the tool reaction and the cutting and separating forces is provided below:

$$Fc = R\cos(\beta - \alpha)$$

$$Ft = R\sin(\beta - \alpha)$$

Where α is the rake angle and β is the friction angle. The friction angle is determined from the following relationship.

$$\beta = ATAN(\mu)$$

The coefficient of friction μ is at the tip of the cutting tool. This value will vary from 0.50 to 2.0.

Using the average values of μ the thrust or separating force can be estimated from the Fc value obtained from energy conservation.

$$\beta = ATAN(1.25) = 51.3^o$$

In the case under consideration the rake angle α is 25^o. Using the value of 367.2 lbs. for Fc, the resultant tool force is obtained.

$$R = \frac{Fc}{\cos(\beta - \alpha)} = 420lbs.$$

The separating force is then estimated from the resultant load.

$$Ft = 420\sin(51.3 - 25) = 186lbs.$$

The thrust force is within 21 percent of the reported value.

These values can now be cross checked with using a SSP model. For nomenclature definitions reference Figure 2.

The shear plane angle needs to be computed. There are two common relationships used to estimate the incline of the shear plane. The first is the Merchant relationship provided below:

$$\phi = 45^o + \frac{\alpha}{2} - \frac{\beta}{2}$$

Another common relationship for the shear angle is Shaffer's:

$$\phi = 45^o + \alpha - \beta$$

The average shear plane angle using Merchant's and Shaffer's relationships is:

$$\phi_{avg} = \frac{(31.8^o + 18.7^o)}{2} = 25.25^o$$

Table 2 outlines the calculations for cross checking the Fc and Ft values obtained from the specific energy calculations:

Table 2 – Single Shear Plane Model

Single Shear Plane Model		
Rank Angle	25.00	**Degrees**
Friction Angle	51.34	**Degrees**
Shear Angle	25.25	**Degrees**
μ	1.25	
Depth of Cut (t_0)	0.0025	**Inches**
Width of Cut (w)	0.48	**Inches**
Ultimate Strength	130	**ksi**
Fs	209	**Lbs.**
Resultant	336	**Lbs.**
Fc	**301**	**Lbs.**
Ft	**149**	**Lbs.**

The SSP model introduces two additional calculations. The first is Fs, which is the ultimate shear load capacity of material on the shear plane. The second is the resultant force based on the maximum shear capacity. For nomenclature definitions, reference Figure 1 and Table 2. The ultimate shear stress is based on the Von Mises stress criterion.

$$Fs = \tau_u \frac{t_o \times w}{\sin\phi}$$

$$\tau_u = \frac{\sigma_u}{\sqrt{3}}$$

The resultant load is calculated from the equation below:

$$R = \frac{Fs}{\cos(\phi + \beta - \alpha)}$$

From the resultant load, Fs and Ft is then calculated. As discussed earlier, the SSP approach tends to provide lower bound values. This is the case with this particular example (e.g., Fc =301 lbs. and Ft = 149 lbs.).

When employing a finite element analysis (FEA) is developing an understanding of the cutting force design space in order to quantify the fault tolerance of a given fixture approach. The variables with the greatest uncertainty and impacting the estimates are friction and the plane of the shear angle.

To understand the design space associated with the tooling forces a Design of Experiments (DOE) should be considered having as its independent variables friction, shear angle, and the material's ultimate strength. A simple spreadsheet can be set up to quickly generate the design space for use in a FEA.

Table 3 – Parameters for Fitting Measured Data

Single Shear Plane Model		
Rank Angle	25.00	**Degrees**
Friction Angle	55.59	**Degrees**
Shear Angle	20.90	**Degrees**
μ	1.46	
Depth of Cut	0.0025	**Inches**
Width of Cut	0.48	**Inches**
Ultimate Strength	143	**ksi**
Fs	275	**Lbs.**
Resultant	441	**Lbs.**
Fc	380	**Lbs.**
Ft	225	**Lbs.**

Table 4 – Design of Experiments
Tool Force Design Space

Full 3 Level DOE			
Run	**μ**	**ϕ**	**σ**
		Deg	ksi
1	1.25	25.25	130
2	1.25	25.25	136
3	2.00	31.8	136
4	1.25	31.8	136
5	0.50	18.7	136
6	0.50	31.8	130
7	2.00	25.25	124
8	1.25	18.7	136
9	2.00	18.7	124
10	0.50	18.7	130
11	2.00	18.7	136
12	2.00	31.8	130
13	0.50	18.7	124
14	0.50	25.25	136
15	2.00	25.25	136
16	0.50	31.8	136
17	2.00	31.8	124
18	0.50	31.8	124
19	1.25	31.8	124
20	2.00	18.7	130
21	1.25	25.25	124
22	0.50	25.25	130
23	0.50	25.25	124
24	2.00	25.25	130
25	1.25	18.7	124
26	1.25	31.8	130
27	1.25	18.7	130

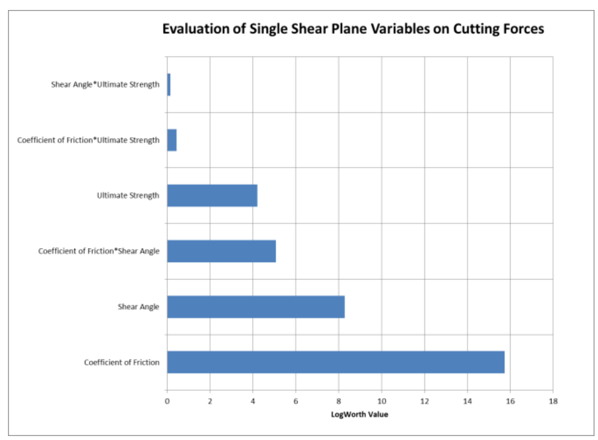

Figure 3 – Evaluation of Shear Plane Variables

The logworth values associated with the DOE Single Shear Plane model variables are provided in Figure 3. A logworth value greater than 2.0 indicates that the variable is significant and should be included in a response surface analysis. In this case all three main effects are significant as well as the cross product of friction and the shear plane angle. Figure 4 provides the response surface of the cutting force data fitted with a 2nd degree polynomial.

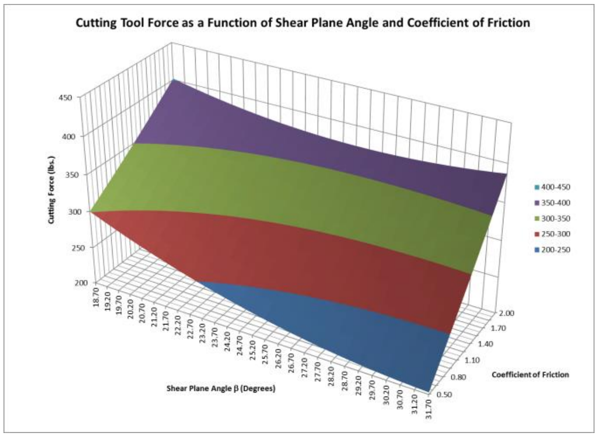

Figure 4 – Cutting Tool Force Response Surface

The separating tool forces are provided in Figure 5. Since the tool rake angle is held constant in this example, the only variable that has significant impact on the separating force response is coefficient of friction between the tool and work piece. It should be noted that at lower friction values the separating forces approach zero. With lower friction values and higher rake angles the possibility of creating negative thrust forces exists. This has important implications in terms of machining stability.

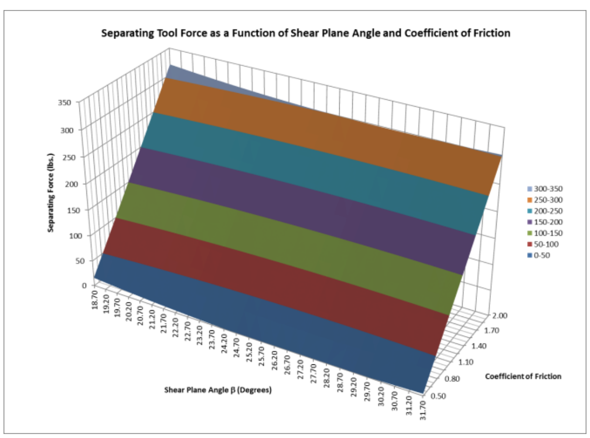

Figure 5 – Separating Tool Force Response Surface

Based on the SSP force analysis the entire domain of the tool force design space can be captured in four runs.

Table 5 – Corner Point Loads of Response Surfaces

Run	Cutting	Separating
	Lbs.	Lbs.
1	300.5	15.9
2	401.6	319.2
3	386.9	302.4
4	202.8	4.1

Conclusions:

As stated in the background section of this brief the Single Shear Plane (SSP) analysis tends to provide a lower bound solution. This is the case where the majority of the space is lower than the actual measured loads. Two of the design space corner points, however, are greater than the actual system forces enabling the response of the fixture system to be evaluated analytically at loads comparable to what would be expected in service.

Using a full factorial experiment with the SSP model is analytically very efficient. The results are obtained from simple spreadsheet calculations. It enables the engineer to see the tool force design space and use the conditions that will potentially limit the design. Only these cases are executed with the finite element model rather than running the entire DOE through a FEA resulting in significant time savings.