Estimating Stress Concentrations with a Minimal Mesh Density Approach

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Abstract:

This tech brief outlines an approach for estimating stress concentrations without employing high mesh density finite element models. The method estimates stress concentrations by evaluating the behavior of the stress intensity or K field in the vicinity of a geometric discontinuity. In components with complex geometry, ensuring that the elastic stress concentration of a geometric discontinuity has actually been captured typically requires several runs with mesh density refinements. The Minimal Mesh Density (MMD) method outlined in this brief is relatively insensitive to mesh density and local geometry features and therefore can be used as a means of quickly cross checking the reasonableness of a single pass stress concentration evaluation. Inherent in the approach is a means of quantitatively evaluating the load transfer characteristics present in the vicinity of the discontinuity. This information can facilitate the identification of design modifications which will tend to mitigate the concentration of stress in the load path. Due to the minimal mesh density and local geometry definition required, this approach can increase the efficiency of load path design studies in areas of complex geometry.

Background:

This Minimal Mesh Density (MMD) approach uses a crack front in the vicinity of the discontinuity as a probe to assess the flow behavior of the stress field around the stress concentration. The stress concentration, at the surface of the discontinuity, is determined from the variation in the stress intensity of an edge crack as it propagates from the notch.

Effective Crack Length

Central to the MMD approach is the concept of the effective crack length created by a geometric discontinuity. Dowling illustrates this well with a crack emanating from a hole in an infinite plate.[1]

[1] *Mechanical Behavior of Materials*, Dowling, Prentice Hall, 1993, p. 299

Three stress intensity solutions are provided for the problem. The first solution computes the stress intensity based on the crack length originating at the center of the hole. The second solution is the actual stress intensity that exists as a crack propagates from the edge of the hole. The third solution is the stress intensity computed for an edge crack with the stress concentration of the hole used to multiple the far-field stress in the stress intensity calculation.

All three solutions come to virtually the same value in the region where the crack length is equal to 10 to 20 percent of the notch radius (i.e., the radius of the hole). As the crack length exceeds 20 percent of the notch radius the third solution, which employs the stress concentration of the notch, begins to significantly deviate from the actual solution. The first solution, however, remains in relatively good agreement with it. The reason for this is as the crack front propagates it is no longer in the influence of the stress concentration of the notch. Using the effective crack length as the sum of the notch radius (ρ) and the crack length (a) provides an answer which is in very good agreement with the actual solution.

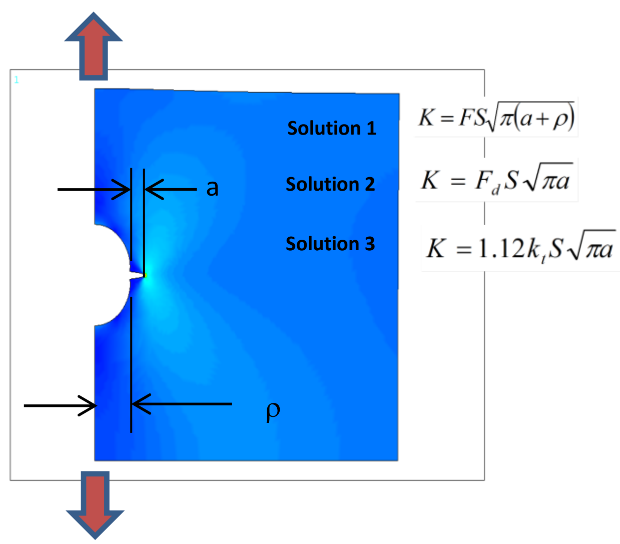

Figure 1 – Half Model of Hole in Infinite Plate

For Solution 1 the form factor F, which is a function of geometry and loading, is equal to 1.0. This value is associated with an edge crack in plane stress under uni-axial loading.

Solution 2 provides the actual stress intensity solution for the problem. The form factor F_d is computed based on the crack length relative to the effective crack length (i.e., the notch radius plus the length of the crack).

Equation 1:

$$d = \frac{a}{(a+\rho)}$$

$$F_d = 0.5(3-d)\left[1+1.243(1-d)^3\right]$$

The form factor for the third solution is 1.12. This is the value for an edge crack in plane strain under uni-axial loading when the crack length is less than 13 percent of the net section.

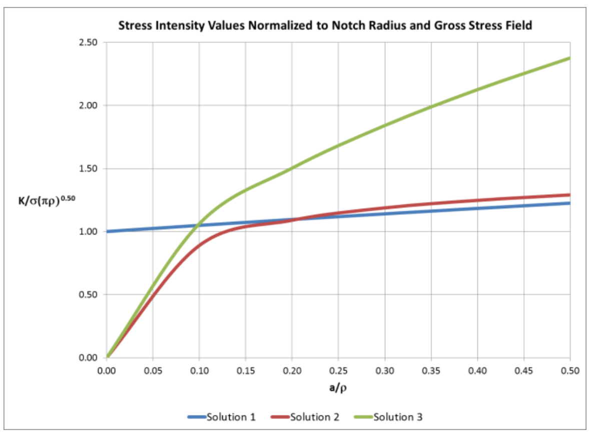

Figure 2 – Normalized Stress Intensity Values

Correlating Stress Intensity and k_t

The concentrated stress in an elliptical flaw is given in equations 2 and 3, where "a" is half the length of the major axis and "ρ" is the radius of curvature at the end of the ellipse and σ is the gross stress field.

Equation 2:

$$k_t = 1 + 2\sqrt{\frac{a}{\rho}}$$

The stress intensity for a plane stress condition can be related to the stress concentration by multiplying the numerator and denominator of the second term of equation 2 by the gross stress field σ and the square root of π.[2] The resulting relationship is given in equation 3.

Equation 3:

$$k_t = 1 + 2\frac{K_I}{\sigma\sqrt{\pi\rho}}$$

Notice in Figure 2, that with Solution 1 the stress intensity values can be well represented by a linear function. Solutions 2 and 3, however, require a power function to correctly capture the behavior within the process zone (i.e., within 10 to 20 percent of the notch radius). Outside the process zone, Solutions 2 and 3 could also be reasonably curve fit with a linear model.

The value for Solution 1 is equal to 1.0 at the surface of the circular hole. In this solution the effective length of the crack originates at the center of the hole. Substituting this value into the quotient of equation 3 the resulting k_t = 3.0. This yields the correct stress concentration value for a hole in an infinite plate.

Estimating k_t with Stress Intensity Values:

The utility gained from approaching the stress intensity calculation using an effective crack length as in Solution 1 is that an effective stress intensity value can be obtained at the surface of the notch without a crack being present. Using the effective stress intensity value an estimate of the notch k_t can then be obtained from equation 3.0.

Crack Front Singular Elements:

In addition to providing a cross check of a converged stress field solution, the advantages of estimating a k_t from stress intensity data are that the mesh density required to obtain the desired information is at a minimum and the actual notch geometry does not need to be present in the model. During a design activity this MMD approach can be more efficient than creating studies with detailed notch geometry changes and models with high mesh densities.

[2] *Peterson's Stress Concentration Factors, 3rd Edition*, Walter and Deborah Pilkey, Wiley, 2008, p.50

In using a coarse mesh to obtain stress intensity values the displacement extrapolation method should be employed. This method uses the displacement field at the tip of singular elements around the crack front for the stress intensity calculation. Relatively coarse meshes can be used to obtain accurate results since the solution's primary field variable is displacement. Figure 3 provides an example of a typical mesh using the singular elements at the crack front.

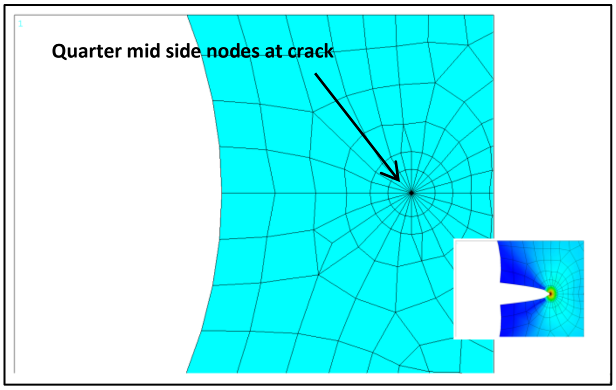

Figure 3- Crack Front Mesh

In ANSYS the stress intensity values are computed in POST 1 using the KCALC command when the displacement extrapolation method is employed. Executing this command requires establishing a local coordinate system at the crack front and defining a path around the crack tip for mapping of the displacement field.[3]

The MMD approach uses edge cracks at the point of the geometric discontinuity where the stress field is interrupted. Edge cracks lend themselves to being swept along complex paths starting with 2D planar elements making the approach easy to implement for detail design studies.

Estimating Process:

In complex geometry it is not always apparent from inspection what is the effective crack length associated with the geometric discontinuity. In the case of a hole in an infinite plate the characteristic dimension for the initial effective crack length (i.e., radius of hole) is apparent. The behavior of Solution 1, however, lends itself to estimating the effective stress intensity at the surface of the notch even when the initial effective crack length is unknown. For Solution 1 the stress intensity behavior as a function of crack growth is relatively linear. This can be used to make an estimate of the effective stress intensity at the notch surface without actually knowing the effective initial crack length.

The process is to first probe the stress field with two or more cracks at different lengths that are beyond the process zone and then extrapolate the stress intensity data back to the notch surface with a linear regression model. As with the example of a hole in an infinite plate, the value can be substituted into equation 3 and the correct stress concentration obtained.

This process also works for a plate edge crack. In this particular case there is no stress concentration at the edge of a plate since no geometric discontinuity exists to disrupt the stress field. When probing the stress field with two or more cracks and using a linear regression model to extrapolate back to the surface the result will be a non-zero value at the surface. When this non-zero value, however, is substituted into equation 3 the correct answer of a k_t = 1.0 is still obtained since the radius of curvature (i.e., ρ) is infinite for a straight edge.

Each of the three solutions in Figure 2 requires a power function to obtain exact stress intensity results. A linear model, however, is very adequate for Solution 1 which employs the concept of an initial effective crack length associated with the feature creating the stress disruption.

Examples of the Process:

The first two examples are stepped bars. The first example has a step ratio (D/d) of 1.5 and the second one of 1.25. In these cases the shoulder of the step can be thought of as the effective length of half the crack face. The other side of the crack face has been eliminated by the step being created. This obviously influences the stress field in the notch radius of the step. The approach outline, however, does not require a priori knowledge of what might be an acceptable effective initial crack length.

In these examples three different crack lengths are employed so that linearity of the probe data can be quantified by the regression analysis of the linear

[3] Reference section 13.3.3.2 of ANSYS Structural Analysis Guide – Version 13

curve fit. In practice only two probes would be necessary. Additionally, the length of the cracks used to probe the stress field have been chosen such that they would be outside the process zone of any notch evaluated in these particular cases. The crack lengths are 0.05, 0.075 and 0.10 inches.

Step Bar Example with D/d = 1.5

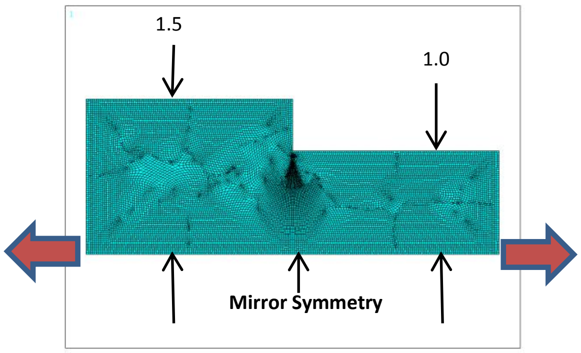

Figure 4 – Step Bar Definition

The mesh density at geometric discontinuity is shown in Figure 5.

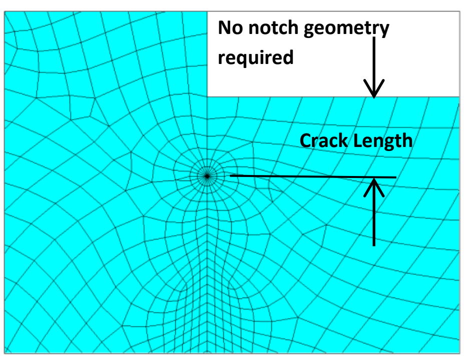

Figure 5- Mesh Density

The gross stress field in this example is 1 ksi and Table 1 provides the results of the crack probe into the stress field at the step.

Table 1 – Stress Intensity Values (Plane Stress)

Crack Length	K_I	K_{II}	K_{eff}
Inches	ksi-in$^{0.50}$	ksi-in$^{0.50}$	ksi-in$^{0.50}$
0.050	0.66	0.12	0.67
0.075	0.72	0.11	0.73
0.100	0.77	0.10	0.77

The values for the in-plane shear stress intensity factor K_{II} are high enough to indicate that the crack probe is not completely perpendicular to the first principle stress. The effective stress intensity values being in good agreement with the K_I values indicates that the probe is adequately aligned with the first principal stress field for use in the stress concentration estimates.

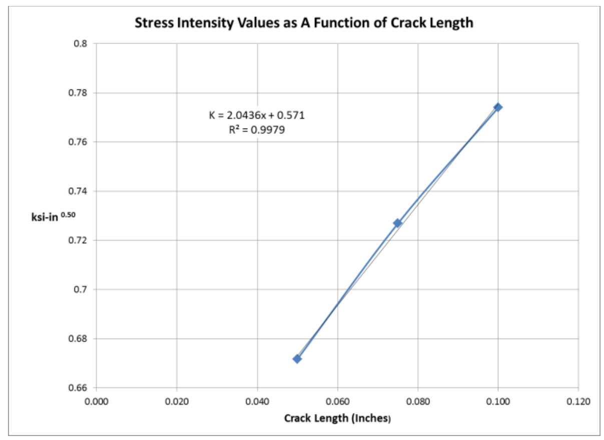

Figure 5 – Linear Regression Model

Figure 6 provides the comparison between the stress concentrations obtained from equation 3 and results from finite element models with converged solutions. The intercept value of 0.571 ksi-in$^{0.50}$ obtained from the linear region is used in equation 3.

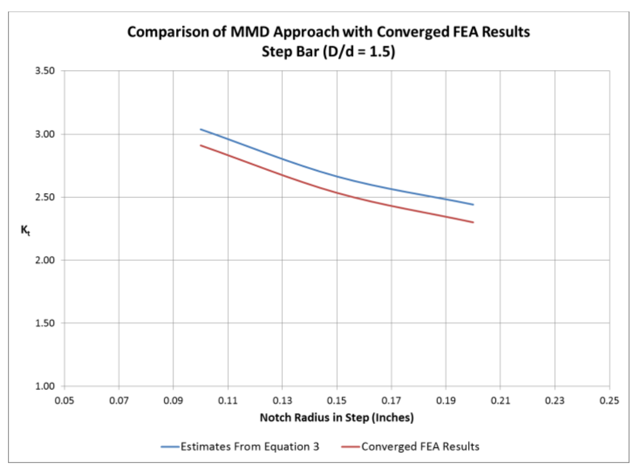

Figure 6

The MMD approach does not require any detailed notch geometry and yet is in good agreement with high density meshed finite element models that explicitly model the notch radius.

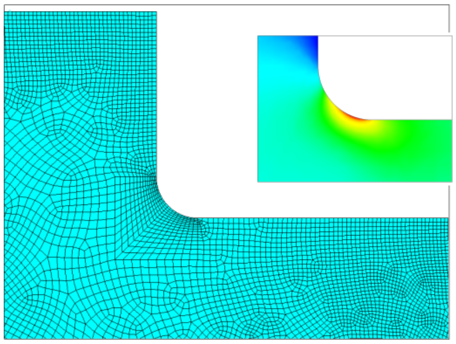

Figure 7 – Mesh Density for Converged Solution

Similar results are obtained with the step bar having a characteristic step ratio of 1.25. Figure 8 provides a plot the comparison.

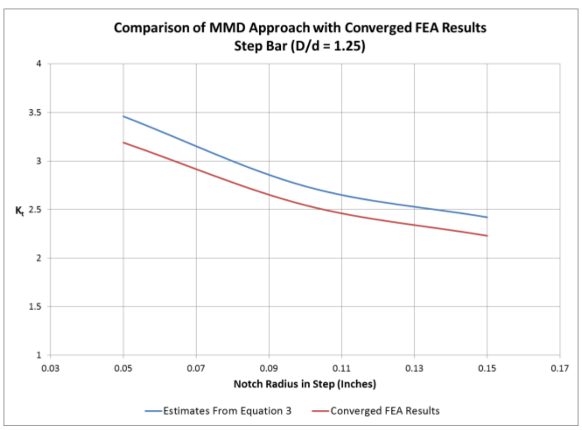

Figure 8

T-Bar Example:

One additional example employing a T-Bar illustrates the need to have the crack probe reasonably aligned with the first principle stress field as well as using plane-strain stress intensity value to provide an upper bound for the MMD approach.

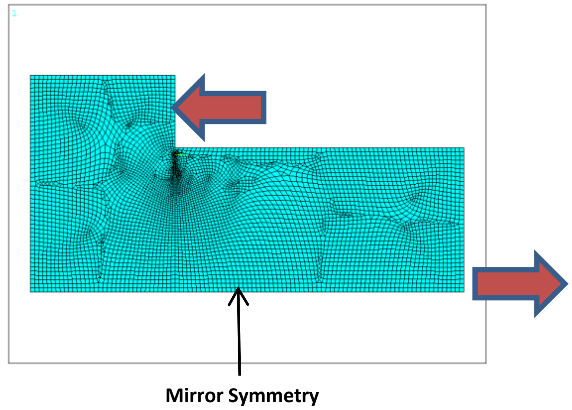

Figure 9: T-Bar

The initial crack probe orientation is shown below.

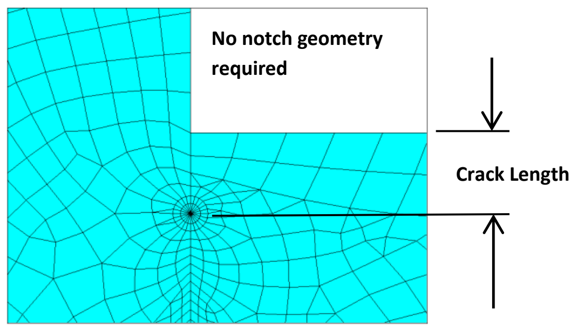

Figure 10: Initial Crack Orientation

Table 2 provides the stress intensity values from the crack probe in the orientation shown in Figure 9. As seen from the data in the table it is clear that the alignment is not perpendicular to the first principal stress field. The in-plane shear is approximately 40 percent of the K_I value. This is due to the load having to flow around the step corner to be equilibrated at the head of the T-Bar.

Table 2 – Stress Intensity Values (Plane Stress)

Crack Length	K_I	K_{II}	K_{eff}
Inches	ksi-in$^{0.50}$	ksi-in$^{0.50}$	ksi-in$^{0.50}$
0.050	1.82	0.70	1.95
0.075	1.87	0.73	2.01
0.100	1.91	0.75	2.05

Figure 11 provides the crack probe orientation that aligns with the first principle stress field at the discontinuity.

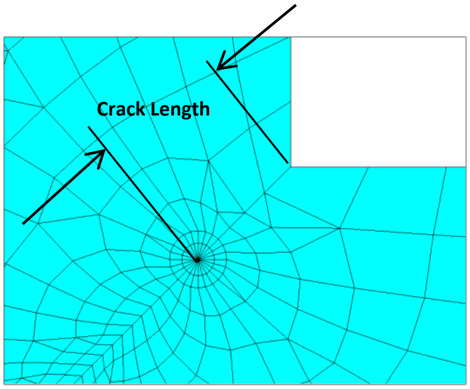

Figure 11: Properly Aligned Crack Orientation

Table 3 provides stress intensity values from the properly aligned crack probe. The in-plane shear contribution is negligible.

Table 3 – Stress Intensity Values (Plane Strain)

Crack Length	K_I	K_{II}	K_{eff}
Inches	ksi-in$^{0.50}$	ksi-in$^{0.50}$	ksi-in$^{0.50}$
0.050	2.54	0.03	2.54
0.075	2.63	0.10	2.63
0.100	2.73	0.10	2.73

In addition to obtaining the stress intensity values aligned with the first principle stress field, the values were also obtained under plane strain conditions. Plane strain conditions provide the maximum stress concentration estimates. Figure 12 is a plot of the T-Bar results.

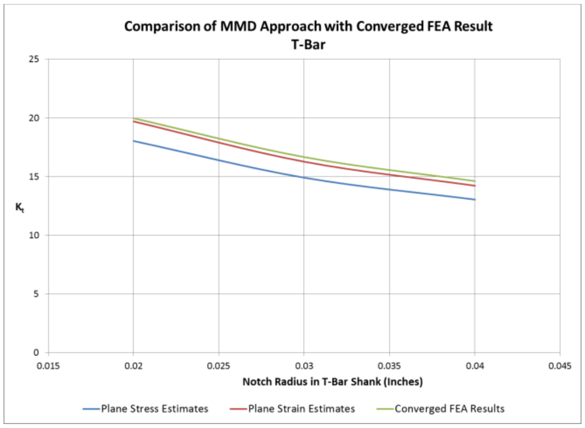

Figure 12

The plane strain stress concentration predictions from equation 3 are within 3 percent of the converged FEA models.

Conclusions:

The MMD approach can provide an efficient means of executing design studies in optimizing geometric features found in complex load paths. Sweeping 2D edge crack models through transition regions of complex geometry makes this approach useful for design optimization. The method requires minimal local geometry features to develop a quantitative understanding of the stress field behavior in the vicinity of geometric discontinuities and minimizes the mesh density required in the studies.

The MMD approach will not always provide conservative results as demonstrated in the T-Bar example. The trend lines, however, will follow a converged solution. Using plane strain stress intensity values will always provide the highest stress concentration estimates for this method.